Formulario

Fundamentos Físicos de la Radiación

Ecuaciones de Maxwell			$\oint_S \vec{D}\cdot d\vec{S} = \int_V \rho dV$	
			$\oint_S \vec{B}\cdot d\vec{S} = 0$	
			$\oint_C \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int_S \vec{B}\cdot d\vec{S}$	
			$\oint_C \vec{H}\cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) d\vec{S}$	
Densidad de Potencia			$\vec{\wp} = \frac{1}{2}\Re e\ \{\vec{E}\times\vec{H}^*\}$	
Polarización	Lineal		$E_x(z,t) = C_{1x}\cos(\omega t - \beta z + \zeta)$ $E_y(z,t) = C_{1y}\cos(\omega t - \beta z + \zeta)$	
	Elíptica	*RHEP*	$E_x(z,t) = C_{1x}\cos(\omega t - \beta z)$ $E_y(z,t) = C_{1y}\cos(\omega t - \beta z - \zeta)$	$E_x(z) = C_{1x}e^{-j\beta z}$ $E_y(z) = C_{1y}e^{-j\beta z}e^{-j\zeta}$
		LHEP	$E_x(z,t) = C_{1x}\cos(\omega t - \beta z)$ $E_y(z,t) = C_{1y}\cos(\omega t - \beta z + \zeta)$	$E_x(z) = C_{1x}e^{-j\beta z}$ $E_y(z) = C_{1y}e^{-j\beta z}e^{+j\zeta}$
	Circular	*RHCP*	$E_x(z,t) = C_1\cos(\omega t - \beta z)$ $E_y(z,t) = C_1\cos\left(\omega t - \beta z - \frac{\pi}{2}\right)$	$\vec{E}(z) = C_1 e^{-j\beta z}\left(\hat{i} - j\,j\right)$
		LHCP	$E_x(z,t) = C_1\cos(\omega t - \beta z)$ $E_y(z,t) = C_1\cos\left(\omega t - \beta z + \frac{\pi}{2}\right)$	$\vec{E}(z) = C_1 e^{-j\beta z}\left(\hat{i} + j\,j\right)$
	Relación axial		$AR = \left(A_{x'}/A_{y'}\right)^{-1}$ $\quad \psi = \frac{1}{2}\tan^{-1}\left[\tan(2\alpha)\cos(\zeta)\right]$ donde $\alpha = \frac{C_{1y}}{C_{1x}}$ $\left.\begin{matrix} A_{x'} = C_{1x}\cos\psi + C_{1y}sen\psi \\ A_{y'} = -C_{1x}sen\psi + C_{1y}\cos\psi \end{matrix}\right\}$ con las constantes relacionadas por $A_{x'}^2 + A_{y'}^2 = C_{1x}^2 + C_{1y}^2$	
Potencial Vector			$\vec{A}(\vec{r}) = \frac{\mu}{4\pi}\int_{v'}\frac{\vec{J}(\vec{r}\,')e^{-jkR}}{R}dv' \approx \frac{\mu}{4\pi}\frac{e^{-jkr}}{r}\int_{v'}\vec{J}(\vec{r}\,')e^{jk\hat{r}\cdot\vec{r}\,'}dv'$ con $\vec{R} = \vec{r} - \vec{r}\,'$	
			$\vec{F} = \frac{\varepsilon}{4\pi}\frac{e^{-jkr}}{r}\int_{v'}\vec{M}(\vec{r}\,')e^{jk\hat{r}\cdot\vec{r}\,'}dV'$	
Vector de Radiación			$\vec{N} = \int_{V'}\vec{J}(\vec{r}\,')e^{jk\hat{r}\cdot\vec{r}\,'}dv' = N_r\hat{r} + N_\theta\hat{\theta} + N_\phi\hat{\phi}$	
			$\vec{L} = \int_{V'}\vec{M}(\vec{r}\,')e^{jk\hat{r}\cdot\vec{r}\,'}dv' = L_r\hat{r} + L_\theta\hat{\theta} + L_\phi\hat{\phi}$	
Relaciones entre el potencial vector, el vector de radiación y los campos de radiación			$\vec{A} = \frac{\mu}{4\pi}\frac{e^{-jkr}}{r}\vec{N}$	
			$E_r = 0 \quad ; \quad H_r = 0$ $E_\theta = -j\omega A_\theta \quad ; \quad H_\theta = j\frac{\omega}{\eta}A_\phi = -\frac{E_\phi}{\eta}$ $E_\phi = -j\omega A_\phi \quad ; \quad H_\phi = -j\frac{\omega}{\eta}A_\theta = \frac{E_\theta}{\eta}$	
			$E_\theta = -j\omega\mu\frac{e^{-jkr}}{4\pi r}\left(N_\theta + \frac{L_\phi}{\eta}\right) = -j\frac{e^{-jkr}}{2\lambda r}\left(\eta N_\theta + L_\phi\right)$ $E_\phi = -j\omega\mu\frac{e^{-jkr}}{4\pi r}\left(N_\phi - \frac{L_\theta}{\eta}\right)N_\phi = -j\frac{e^{-jkr}}{2\lambda r}\left(\eta N_\phi - L_\theta\right)$	
Regiones de interés	Fraunhofer		$2\frac{D^2}{\lambda} \le r < \infty$	
	Fresnel		$0.6\left(\frac{D^3}{\lambda}\right)^{\frac{1}{2}} \le r \le \frac{2D^2}{\lambda}$	
	Cercano reactivo		$r < 0.6\left(\frac{D^3}{\lambda}\right)^{\frac{1}{2}}$	

Parámetros de Antenas

Impedancia de entrada	$Z_a(\omega)=V_{ent}/I_{ent}=R_a(\omega)+jX_a(\omega)$	Resistencia de antena	$R_a=R_{rad}+R_{\Omega}$
Potencia radiada y resistencia de radiación	$P_{radiada}=\frac{1}{2}\lvert I_a\rvert^2 R_{rad}\Rightarrow R_{rad}=2\frac{P_{rad}}{\lvert I_a\rvert^2}$	Coeficiente de Reflexión	$\Gamma=S_{11}=\frac{Z_a-Z_0}{Z_a+Z_0}=\frac{Z_a/Z_0-1}{Z_a/Z_0+1}$
Pérdida de retorno y ROE	$RL=10\log\frac{P_{ref}}{P_{inc}}=20\log(\Gamma)$ [dB] , $ROE=VSWR=\frac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}$	Eficiencia de antena	$e_f=\frac{R_{rad}}{R_{rad}+R_{\Omega}}$
Resistencia óhmica	Resistencia de Continua: $R_{dc}=\frac{1}{\sigma}\frac{l}{A},[\Omega]$	Intensidad de Radiación	$K(\theta,\phi)=\wp(\theta,\phi)r^2$ [W/sr]
	Resistencia de alta frecuencia: $R_{hf}=R_s\frac{l}{P}$ con $R_s=\sqrt{\frac{\pi f\mu}{\sigma}},[\Omega]$	ángulo sólido de haz	$\Omega_A=\int_0^{2\pi}\int_0^{\pi}t_n(\theta,\phi)\,sen\theta\,d\theta\,d\phi$
Ganancia directiva	$D_g(\theta,\phi)=D(\theta,\phi)=\frac{\wp(\theta,\phi)}{P_r/(4\pi r^2)}$	Directividad	$D=\frac{\wp_{máx}}{P_r/(4\pi r^2)}=\frac{\wp_{máx}}{\frac{\iint\wp(\theta,\phi)r^2 sen(\theta)d\theta d\phi}{4\pi r^2}}$
Formula de Kraus	$D=\frac{4\pi}{\Omega_A}\simeq\frac{4\pi}{\theta_{HP}\phi_{HP}}$ (ángulos en radianes) ; $D\simeq\frac{41253}{\theta_{HP}\phi_{HP}}$ (ángulos en grados)	Formula de Tai-Pereira	$D\simeq\frac{32\ln 2}{(\theta_{HP})^2+(\phi_{HP})^2}$ (ángulos en radianes)
Ganancia de antena	$G(\theta,\phi)=e_f D_g(\theta,\phi)$	PIRE	$PIRE=G\cdot P_{entregada}=D_g\cdot P_{radiada}$
Coeficiente de desadaptación	$C_a=\frac{4R_aR_L}{(R_a+R_L)^2+(X_a+X_L)^2}$	Eficiencia de apertura	$e_{ap}=\frac{A_{ef}}{A}$
Área efectiva	$A_{ef}=\frac{P_{recibida}}{\wp_{incidente}}=\frac{\lvert V_{CA}\rvert^2}{4R_a\wp_{incidente}}=\frac{\lvert V_{CA}\rvert^2\eta}{4R_a\lvert E_{inc}\rvert^2}\Rightarrow A_{ef_{máx}}=D\frac{\lambda^2}{4\pi}$	Longitud efectiva	$l_{ef}=\frac{\lvert V_{CA}\rvert}{\lvert E_{inc}\rvert}=\frac{1}{I_0}\left(\vec{N}-(\vec{N}\cdot\hat{r})\hat{r}\right)$
Relación Señal- ruido	$\frac{S}{N_{DR}}=\frac{P_L}{P_{NDR}}=\frac{P_{rad}G_TA_{efR}}{4\pi r^2kT_AB}=\frac{P_{rad}G_T\lambda^2}{(4\pi r)^2kB}\left(\frac{G_R}{T_A}\right)$	Área-longitud efectiva	$A_{ef}=\frac{l_{ef}^2\eta}{4R_a}$

Antenas Lineales y de Lazo

Dipolo eléctrico elemental	Campo de radiación	$E_\theta = j\omega\mu_0 \frac{I_0 L}{4\pi}\frac{e^{-jkr}}{r} sen\theta$ $H_\phi = jk\frac{I_0 L}{4\pi}\frac{e^{-jkr}}{r} sen\theta$
	Densidad de potencia	$\vec{\wp} = \left(\frac{I_0 L}{4\pi r}\right)^2 \omega\mu_0 k \; \text{sen}^2\theta \; \hat{r}$
	Potencia radiada	$P_{rad} = \frac{(I_0 L)^2}{6\pi}\omega\mu_0 k$
	Resistencia de radiación	$R_{rad} = 80\pi^2\left(\frac{L}{\lambda}\right)^2 \quad [\Omega]$
	Longitud efectiva	$\vec{l}_{ef} = -Lsen\theta \; \hat{\theta}$
	Área efectiva máxima	$A_{ef_{máx}} = \frac{3}{8}\frac{\lambda^2}{\pi}$
	Intensidad de Radiación	$K(\theta,\phi) = \frac{I_0^2\eta}{4}\left(\frac{L}{\lambda}\right)^2 \text{sen}^2\theta$

Dipolo magnético elemental	Campo de radiación	$E_\phi = \frac{\omega\mu_0 km}{4\pi r} sen\theta e^{-jkr}$ $H_\theta = -\frac{\omega\mu_0 km}{4\pi\eta r} sen\theta e^{-jkr}$
	Densidad de potencia	$\vec{\wp} = \hat{r}\left(\frac{\omega\mu_0 km}{4\pi r\sqrt{\eta}}\right)^2 \text{sen}^2\theta$
	Potencia radiada	$P_{rad} = \frac{(\omega\mu_0 km)^2}{6\pi\eta}$
	Resistencia de radiación	$R_{rad} = 320\pi^6\left(\frac{a}{\lambda}\right)^4 \quad [\Omega]$
	Longitud efectiva	$\vec{l}_{ef} = jk\pi a^2 \text{sen}\theta \; \hat{\phi}$
	Área efectiva máxima	$A_{ef_{máx}} = \frac{3}{8}\frac{\lambda^2}{\pi}$
	Intensidad de Radiación	$K(\theta,\phi) = \frac{\omega^2\mu_0^2 m^2}{4\lambda^2\eta} \text{sen}^2\theta$

Dipolo eléctrico corto	Campo de radiación	$E_\theta = j\eta\frac{kI_0 Le^{-jkr}}{8\pi r} sen\theta$ $H_\phi = j\frac{kI_0 Le^{-jkr}}{8\pi r} sen\theta$
	Densidad de potencia	$\vec{\wp} = \frac{\eta k^2 I_0^2 L^2 \text{sen}^2\theta}{128\pi^2 r^2}$
	Potencia radiada	$P_{rad} = 10\pi^2 I_0^2\left(\frac{L}{\lambda}\right)^2$
	Resistencia de radiación	$R_{rad} = 20\pi^2\left(\frac{L}{\lambda}\right)^2 \quad [\Omega]$
	Longitud efectiva	$l_{ef} = \frac{L}{2}$
	Área efectiva máxima	$A_{ef_{máx}} = \frac{3}{2}\frac{\lambda^2}{\pi}$
	Intensidad de Radiación	$K(\theta,\phi) = \frac{\eta k^2 I_0^2 L^2}{128\pi^2}\text{sen}^2\theta$

Dipolo eléctrico arbitrario	Campo de radiación	$E_\theta = j\eta\frac{e^{-jkr}}{2\pi r}I_0\frac{\cos\left(k\frac{L}{2}\cos\theta\right)-\cos\left(k\frac{L}{2}\right)}{sen\theta}$ $H_\phi = \frac{E_\theta}{\eta} = j\frac{e^{-jkr}}{2\pi r}I_0\frac{\cos\left(k\frac{L}{2}\cos\theta\right)-\cos\left(k\frac{L}{2}\right)}{sen\theta}$
	Densidad de potencia	$\vec{\wp} = \frac{\eta}{8\pi^2 r^2}I_0^2\frac{\left[\cos\left(k\frac{L}{2}\cos\theta\right)-\cos\left(k\frac{L}{2}\right)\right]^2}{sen^2\theta}\hat{r}$
	Resistencia de radiación	$R_{rad} = \frac{2P_{rad}}{I_0^2} = \frac{2}{I_0^2}\frac{\eta I_0^2}{4\pi}\int_0^\pi \frac{\left[\cos\left(k\frac{L}{2}\cos\theta\right)-\cos\left(k\frac{L}{2}\right)\right]^2}{sen\theta}d\theta =$ $= \frac{\eta}{2\pi}\left\{C + Ln(kL) - C_i(kL) + \frac{1}{2}sen(kL)\left[S_i(2kL) - 2S_i(kL)\right] +\right.$ $\left. + \frac{1}{2}\cos(kL)\left[C + Ln\left(\frac{kL}{2}\right) + C_i(2kL) - 2C_i(kL)\right]\right\}$
	Resistencia de antena	$R_a = \frac{\eta}{2\pi sen(kL/2)}\left\{C + Ln(kL) - C_i(kL) + \frac{1}{2}sen(kL)\left[S_i(2kL) - 2S_i(kL)\right] +\right.$ $\left. + \frac{1}{2}\cos(kL)\left[C + Ln\left(\frac{kL}{2}\right) + C_i(2kL) - 2C_i(kL)\right]\right\}$
	Reactancia de antena	$X_a = 30\left\{2S_i(kL) + \cos(kL)\left[2S_i(kL) - S_i(2kL)\right] -\right.$ $\left. -sen(kL)\left[2C_i(kL) - C_i(2kL) - C_i\left(\frac{2Ka^2}{L}\right)\right]\right\}$
	Longitud efectiva	$\vec{l}_{ef} = \left(\frac{\lambda}{\pi}\right)\frac{\cos\left(k\frac{L}{2}\cos\theta\right)-\cos\left(k\frac{L}{2}\right)}{sen\left(k\frac{L}{2}\right)sen\theta}\hat{\theta}$
	Intensidad de Radiación	$K(\theta,\phi) = \frac{\eta}{8\pi^2}I_0^2\frac{\left[\cos\left(k\frac{L}{2}\cos\theta\right)-\cos\left(k\frac{L}{2}\right)\right]^2}{sen^2\theta}$

Dipolo resonante	Campo de radiación	$E_\theta = 60j\frac{e^{-jkr}}{r}I_0\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{sen\theta}$ $H_\phi = \frac{E_\theta}{\eta} = j\frac{e^{-jkr}}{2\pi r}I_0\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{sen\theta}$
	Densidad de potencia	$\vec{\wp} = \frac{\eta}{8\pi^2 r^2}I_0^2\frac{\left[\cos\left(\frac{\pi}{2}\cos\theta\right)\right]^2}{sen^2\theta}\hat{r}$
	Potencia radiada	$P_{rad} = 36.525 I_0^2$
	Longitud efectiva	$\vec{l}_{ef}\Big\rvert_{\theta=\pi/2} = \frac{\lambda}{\pi}\hat{\theta}$
	Intensidad de Radiación	$K(\theta,\phi) = \frac{\eta}{8\pi^2}I_0^2\frac{\left[\cos\left(\frac{\pi}{2}\cos\theta\right)\right]^2}{sen^2\theta}$
	Impedancia de antena	$Z_a = 73.1 + j42.5 \quad [\Omega]$

Impedancia mutua	$\left.\begin{array}{l} V_1 = I_1(0)Z_{11} + I_2(0)Z_{12} \\ V_2 = I_1(0)Z_{21} + I_2(0)Z_{22} \end{array}\right\}$
	$Z_{11} = \frac{V_1}{I_1(0)}\Big\rvert_{I_2=0} = -\frac{1}{I_1^2(0)}\int I_1(z)E_{11}dz$ $Z_{12} = -\frac{1}{I_1(0)I_2(0)}\int I_1(z)E_{12}dz = Z_{21}$

Dipolo vertical frente a plano a tierra	Campo de radiación	$E_\theta = j\eta\frac{kI_0 Le^{-jkr}}{4\pi r}sen\theta\left[2\cos(kh\cos\theta)\right]$
	Potencia radiada	$P_{rad} = \pi\eta\left(\frac{I_0 L}{\lambda}\right)^2\left[\frac{1}{3} - \frac{\cos(2kh)}{(2kh)^2} + \frac{sen(2kh)}{(2kh)^3}\right]$
	Directividad	$D = \frac{2}{\left[\frac{1}{3} - \frac{\cos(2kh)}{(2kh)^2} + \frac{sen(2kh)}{(2kh)^3}\right]}$
	Resistencia de radiación	$R_{rad} = 2\pi\eta\left(\frac{L}{\lambda}\right)^2\left[\frac{1}{3} - \frac{\cos(2kh)}{(2kh)^2} + \frac{sen(2kh)}{(2kh)^3}\right]$

vertical frente a plano a	Campo de radiación	$E_\theta = j\eta\frac{kI_0 Le^{-jkr}}{4\pi r}\sqrt{1 - sen^2\theta sen^2\phi}\left[2jsen(kh\cos\theta)\right]$
	Potencia radiada	$P_{rad} = \eta\frac{\pi}{2}\left(\frac{I_0 L}{\lambda}\right)^2\left[\frac{2}{3} - \frac{sen(2kh)}{(2kh)} - \frac{\cos(2kh)}{(2kh)^2} + \frac{sen(2kh)}{(2kh)^3}\right]$

<table>
<tr><td rowspan="2"></td><td colspan="2">Directividad</td><td colspan="4">$D=\begin{cases}\dfrac{4sen^2(kh)}{R(kh)} & \forall kh \le \pi/2 \ \ (h \le \lambda/4)\\ \dfrac{4}{R(kh)} & \forall kh > \pi/2 \ \ (h > \lambda/4)\end{cases}$ donde $R(kh)=\left[\dfrac{2}{3}-\dfrac{sen(2kh)}{(2kh)}-\dfrac{\cos(2kh)}{(2kh)^2}+\dfrac{sen(2kh)}{(2kh)^3}\right]$</td></tr>
<tr><td colspan="2">Resistencia de radiación</td><td colspan="4">$R_{rad}=\eta\pi\left(\dfrac{L}{\lambda}\right)^2\left[\dfrac{2}{3}-\dfrac{sen(2kh)}{(2kh)}-\dfrac{\cos(2kh)}{(2kh)^2}+\dfrac{sen(2kh)}{(2kh)^3}\right]$</td></tr>
<tr><td rowspan="6">Antena circular de corriente constante</td><td colspan="2">Campo de radiación</td><td>$E_\phi=\dfrac{a\eta kI_0e^{-jkr}}{2r}J_1(kasen\theta)$
$H_\theta=-\dfrac{akI_0e^{-jkr}}{2r}J_1(kasen\theta)$</td><td colspan="3" rowspan="4">Aproximación de radio medio (λ/6π< a <λ/2)
No hay aproximaciones</td></tr>
<tr><td rowspan="5">Aproximación de gran radio
(a >λ/2)</td><td>Densidad de potencia</td><td>$\vec{\wp}=\hat{r}\dfrac{(a\omega\mu)^2 I_0^2}{8\eta r^2}J_1^2(kasen\theta)$</td></tr>
<tr><td>Intensidad de radiación</td><td>$\vec{K}(\theta,\phi)=\dfrac{(a\omega\mu)^2 I_0^2}{8\eta}J_1^2(kasen\theta)\hat{r}$</td></tr>
<tr><td>Potencia radiada</td><td>$P_{rad}=\dfrac{\pi(a\omega\mu)^2 I_0^2}{4\eta(ka)}$</td></tr>
<tr><td>Resistencia de radiación</td><td>$R_{rad}=60\pi^2\left(\dfrac{C}{\lambda}\right)$</td><td colspan="3" rowspan="2">Aproximación de radio pequeño (a<λ/6π)
Coincide con el dipolo magnético elemental</td></tr>
<tr><td>Directividad</td><td>$D=0.682\left(\dfrac{C}{\lambda}\right)$</td></tr>
<tr><td rowspan="4">Antena de hélice</td><td rowspan="2">Modo Broadside</td><td rowspan="1">Campo de radiación</td><td>$\vec{E}=\left(j\eta\dfrac{kI_0Se^{-jkr}}{4\pi r}sen\theta\right)\hat{\theta}+\left(\eta\dfrac{k^2\left(\frac{D}{2}\right)^2 I_0e^{-jkr}}{4r}sen\theta\right)\hat{\phi}$</td><td rowspan="4">Modo Endfire
(Válidas para 12º < α < 14º y ¾ y C/λ<4/3 con N >3)</td><td>Campo de radiación

Impedancia de entrada</td><td>$E=sen\left(\dfrac{\pi}{2N}\right)\cos\theta\dfrac{sen\left[\left(\frac{N}{2}\right)\Psi\right]}{sen\left(\frac{\Psi}{2}\right)}$
con
$\Psi=K_0\left(S\cos\theta-\dfrac{L_0}{p}\right)$
$p=\dfrac{L_0/\lambda_0}{S/\lambda_0+1}$
$Z_{ent}=R=140\left(\dfrac{C}{\lambda}\right)$</td></tr>
<tr><td rowspan="3">Relación axial</td><td rowspan="3">$AR=\dfrac{2\lambda S}{(\pi D)^2}$</td><td>Ancho de haz a -3 dB</td><td>$HPBW=\dfrac{52\lambda^{\frac{3}{2}}}{C\sqrt{NS}}(en\ grados)$</td></tr>
<tr><td>Directividad</td><td>$D=15N\dfrac{C^2S}{\lambda^3}$</td></tr>
<tr><td>Relación axial</td><td>$AR=\dfrac{2N+1}{2N}$</td></tr>
</table>

Agrupaciones de Antenas

Principio de Multiplicación de diagramas	$\vec{N}(\vec{r}) = \vec{N}_0(\vec{r})\sum_{n=0}^{N-1} I_n e^{j\omega_z n}$
Margen visible sobre eje Z	$[-kd+\alpha, kd+\alpha]$
Variable compleja	$z = e^{j\Psi}$

Frecuencia digital y ángulo eléctrico	Sobre eje X	$\omega_x = k_x d = kdsen\theta\cos\phi$ $\Psi_x = kdsen\theta\cos\phi + \alpha$
	Sobre eje Y	$\omega_y = k_y d = kdsen\theta sen\phi$ $\Psi_y = kdsen\theta sen\phi + \alpha$
	Sobre eje Z	$\omega_z = k_z d = kd\cos\theta$ $\Psi_z = kd\cos\theta + \alpha$
Máximos y mínimos		$\theta_{máx} = \arccos\left(\frac{2m\pi - \alpha}{kd}\right)$
		$\theta_{\min} = \arccos\left(\frac{(2m+1)\pi - \alpha}{kd}\right)$
Polinomio de la agrupación		$P(z) = \sum_{n=0}^{N-1} a_n z^n = a_0 + a_1 z + a_2 z^2 + \cdots + a_{N-1} z^{N-1}$

Distribución de corriente uniforme	Polinomio de la agrupación	$P(z) = \sum_{n=0}^{N-1} z^n = 1 + z + z^2 + \cdots + z^{N-1} = \frac{z^N - 1}{z-1}$
	Factor de array normalizado	$\lvert FA(\Psi)\rvert = \frac{\left\lvert sen\left(N\frac{\Psi}{2}\right)\right\rvert}{N\left\lvert sen\left(\frac{\Psi}{2}\right)\right\rvert}$
	Ceros y ancho de haz en el espacio eléctrico	$\Psi_c = n\frac{2\pi}{N} \quad \Delta\Psi_c = \frac{4\pi}{N}$
	NLPS	$NLPS = N\left\lvert sen\left(\frac{3\pi}{2N}\right)\right\rvert$
	Ancho a -3dB	$\Delta\theta_{-3dB} = arc\cos\left(\cos\theta_{\max} - 0.44\frac{\lambda}{Nd}\right) - arc\cos\left(\cos\theta_{\max} + 0.44\frac{\lambda}{Nd}\right)$
	Broadside	$\Delta\theta_c \approx 2\left(\frac{\lambda}{Nd}\right) = 2\frac{\lambda}{L} \quad \Delta\theta_{-3dB} \approx 2\left(0.44\frac{\lambda}{Nd}\right) = 0.88\frac{\lambda}{L}$
	Endfire	$\Delta\theta_c = \sqrt{8\frac{\lambda}{Nd}} = \sqrt{8\frac{\lambda}{L}} \quad \Delta\theta_{-3dB} = \sqrt{3.5\frac{\lambda}{Nd}} = \sqrt{3.5\frac{\lambda}{L}}$

Distribución de corriente triangular	Polinomio de la agrupación	$P(z) = \left[\sum_{n=0}^{\frac{N-1}{2}} z^n\right]^2 = \left[\frac{z^{\frac{N+1}{2}} - 1}{z-1}\right]^2$
	Factor de array normalizado	$FA(\Psi) = \frac{\left\lvert sen\left(\frac{N+1}{4}\Psi\right)\right\rvert^2}{N\left\lvert sen\left(\frac{\Psi}{2}\right)\right\rvert^2}$
	Ancho de haz en el espacio eléctrico	$\Delta\Psi_c = \frac{8\pi}{N+1}$
	NLPS	$NLPS = \left(\frac{N+1}{2}\right)^2 \left\lvert sen\left(\frac{3\pi}{N+1}\right)\right\rvert^2$
Distribución de corriente Binómica	Polinomio de la agrupación	$P(z) = (z+1)^{N-1}$
	Factor de array	$\lvert FA(\Psi)\rvert = \left\lvert 2\cos\left(\frac{\Psi}{2}\right)\right\rvert^{N-1}$

Directividad	$D = \frac{\left(\sum_{n=0}^{N-1} a_n\right)^2}{\sum_{n=0}^{N-1} a_n^2 + \sum_{n=0}^{N-2}\sum_{q=n+1}^{N-1} 2a_n a_q \frac{sen\left[kd(n-q)\right]}{kd(n-q)}\cos\left((n-q)\alpha\right)}$

Agrupación de Hansen- Woodyard	Condición de Hansen	$\alpha = -kd - \frac{2.94}{N} \approx -kd - \frac{\pi}{N}$
	Ancho de haz a -3dB	$\Delta\theta_{-3dB} = 2arcsen\left(\sqrt{0.28\frac{\lambda}{Nd}}\right) \approx \sqrt{1.11\frac{\lambda}{Nd}}$
	Ancho entre ceros	$\Delta\theta_c = 2arcsen\left(\sqrt{\frac{\lambda}{Nd}}\right) \simeq 2\sqrt{\frac{\lambda}{Nd}}$
	Directividad	$D = 7.2\frac{Nd}{\lambda}$

Agrupaciones bidimensionales	Factor de Array	$FA(\omega_x, \omega_y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} I_{mn} e^{jn\omega_x} e^{jm\omega_y}$
	Frecuencia digital	$\omega_x = k_x d_x = kd_x sen(\theta)\cos(\phi)$
		$\omega_y = k_y d_y = kd_y sen(\theta) sen(\phi)$
	Máximos	$\theta_{\max} = arcsen\sqrt{\left(\frac{\alpha_x}{kd_x}\right)^2 + \left(\frac{\alpha_y}{kd_y}\right)^2}$ $\phi_{\max} = \arctan\left(\frac{d_x\alpha_y}{d_y\alpha_x}\right)$

Síntesis de Schelkunoff	Polinomio de la agrupación	Número par de ceros	$P(z) = \prod_{n=1}^{\frac{N-1}{2}}\left(z^2 - 2z\cos\psi_{cn} + 1\right)$
		Número impar de ceros	$P(z) = (z\pm1)\prod_{n=1}^{\frac{N-1}{2}}\left(z^2 - 2z\cos\psi_{cn} + 1\right)$
Series de Fourier	Coeficiente de Fourier	$a_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} F_A(\psi)e^{-jn\psi}d\psi = \frac{1}{\pi}\int_0^{\pi} F_A(\psi)\Big\rvert_{par}\cos(n\psi)d\psi - \frac{j}{\pi}\int_0^{\pi} F_A(\psi)\Big\rvert_{impar} sen(n\psi)d\psi$	

Síntesis de Dolph-Chebyshev	Polinomios de Chebyshev	$T_0(x) = 1$ $T_1(x) = x$ $T_2(x) = 2x^2 - 1$ $T_3(x) = 4x^3 - 3x$ $T_4(x) = 8x^4 - 8x^2 + 1$
	Transformación de Dolph	$x = x_0\cos\left(\frac{\psi}{2}\right)$
		$x_0 = \cosh\left(\frac{\cosh^{-1}(NLPS)}{n}\right)$

Aperturas, bocinas y Reflectores

Apertura con campo iluminante			
Corrientes equivalentes	Eléctricas	$\vec{J}_s = \hat{n}\times\vec{H}$	
	Magnéticas	$\vec{M}_s = -\hat{n}\times\vec{E}$	
Vectores de radiación	Eléctricas	$N_x = \iint_{S_0}\left(-\frac{E_x}{Z_0}\right)e^{jk_x x'}e^{jk_y y'}dx'dy'$	
	Magnéticas	$L_y = \iint_{S_0}(-E_x)e^{jk_x x'}e^{jk_y y'}dx'dy'$	
Campos		$E_\theta = j\frac{e^{-jkr}}{2\lambda r}\left(1+\frac{\eta}{Z_0}\cos\theta\right)\cos\phi\iint_{S_0}E_x e^{jk_x x'}e^{jk_y y'}dx'dy'$	
		$E_\phi = -j\frac{e^{-jkr}}{2\lambda r}\left(\frac{\eta}{Z_0}+\cos\theta\right)sen\phi\iint_{S_0}E_x e^{jk_x x'}e^{jk_y y'}dx'dy'$	
Directividad		$D = \frac{4\pi}{\lambda^2}\frac{\left\lvert\iint_{S_0}E_x(x',y')dx'dy'\right\rvert^2+\left\lvert\iint_{S_0}E_y(x',y')dx'dy'\right\rvert^2}{\iint_{S_0}\left(\lvert E_x(x',y')\rvert^2+\lvert E_y(x',y')\rvert^2\right)dx'dy'}$	
Área efectiva		$A_{ef} = A_{geom}\eta_{il}$	
Eficiencia de iluminación		$\eta_{il} = \frac{1}{S_0}\frac{\left\lvert\iint_{S_0}E_x(x',y')dx'dy'\right\rvert^2+\left\lvert\iint_{S_0}E_y(x',y')dx'dy'\right\rvert^2}{\iint_{S_0}\left(\lvert E_x(x',y')\rvert^2+\lvert E_y(x',y')\rvert^2\right)dx'dy'}$	

Apertura elemental	
Campos	$E_\theta = j\frac{e^{-jkr}}{2\lambda r}(1+\cos\theta)E_0\cos\phi\,\Delta S$
	$E_\phi = -j\frac{e^{-jkr}}{2\lambda r}(1+\cos\theta)E_0 sen\phi\,\Delta S$
Intensidad de radiación	$K = \frac{E_0^2(\Delta S)^2}{\eta\lambda^2}\cos^4\left(\frac{\theta}{2}\right)$
Potencia Radiada	$P_{rad} = \frac{4\pi}{3}\frac{E_0^2(\Delta S)^2}{\lambda^2\eta}$

Apertura Rectangular uniforme	
Corrientes equivalentes	$\vec{J}_s = \hat{n}\times\vec{H} = -\frac{E}{Z_0}\hat{x}$ $\vec{M}_s = -\hat{n}\times\vec{E} = -E\hat{y}$
Campo	$\vec{E}(r,\theta,\phi) = \frac{jkE}{4\pi r}ab(1+\cos\theta)e^{-jkr}\left[\frac{sen\left(\frac{ka}{2}sen\theta\cos\phi\right)}{\left(\frac{ka}{2}sen\theta\cos\phi\right)}\right]\left[\frac{sen\left(\frac{kb}{2}sen\theta sen\phi\right)}{\left(\frac{kb}{2}sen\theta sen\phi\right)}\right](\cos\phi\hat{\theta}-sen\phi\hat{\phi})$
Directividad	$D = \frac{4\pi ab}{\lambda^2}$

Apertura iluminada con el modo TE_{10}	
Corrientes equivalentes	$\vec{M}_s = -\hat{n}\times\vec{E}_1 = \hat{x}\,E_0 sen\left(\frac{\pi x}{a}\right)$
	$\vec{J}_s = -\hat{y}\,\frac{E_0}{Z_0}sen\left(\frac{\pi x}{a}\right)$
Campos	$E_\theta = j\frac{abE_0e^{-jkr}}{2\lambda r}\left(1+\frac{\eta}{Z_0}\cos\theta\right)\left(\frac{\pi}{2}\right)sen\phi\left(\frac{\cos\left(k\frac{a}{2}sen\theta\cos\phi\right)}{\left(\frac{\pi}{2}\right)^2-\left(k\frac{a}{2}sen\theta\cos\phi\right)^2}\right)\left(\frac{sen\left(k\frac{b}{2}sen\theta sen\phi\right)}{k\frac{b}{2}sen\theta sen\phi}\right)$
	$E_\phi = j\frac{abE_0e^{-jkr}}{2\lambda r}\left(\cos\theta+\frac{\eta}{Z_0}\right)\left(\frac{\pi}{2}\right)\cos\phi\left(\frac{\cos\left(k\frac{a}{2}sen\theta\cos\phi\right)}{\left(\frac{\pi}{2}\right)^2-\left(k\frac{a}{2}sen\theta\cos\phi\right)^2}\right)\left(\frac{sen\left(k\frac{b}{2}sen\theta sen\phi\right)}{k\frac{b}{2}sen\theta sen\phi}\right)$
Directividad	$D = 10.2\frac{ab}{\lambda^2}$

Bocinas

Bocina plano H	
Diferencia de fases	$K\delta = K\frac{x^2}{2L_H}$
Campo iluminante	$\vec{E} = \hat{j}E_0\cos\left(\frac{\pi}{A}x\right)e^{-j\beta\frac{x^2}{2L_H}}$
Parámetros óptimos	$A_{opt} = \sqrt{3\lambda L_H}$ $\Delta\theta^E_{-3dB} = 50\frac{\lambda}{b}(grados)$; $\Delta\theta^H_{-3dB} = 78\frac{\lambda}{A}(grados)$ $NLPS^E = 13dB$; $NLPS^H = 12dB$ $D_H = 7.9\frac{Ab}{\lambda^2}$: $\eta_{il} = 0.62$

Piramidal
$D_{piramidal} = \frac{\pi}{32}\left(D_E\frac{\lambda}{a}\right)\left(D_H\frac{\lambda}{b}\right) = \frac{\pi\lambda^2}{32ab}D_ED_H$ $\Delta\theta^E_{-3dB} = 56\frac{\lambda}{B}(grados)$; $\Delta\theta^H_{-3dB} = 78\frac{\lambda}{A}(grados)$ $NLPS^E = 10\ dB$; $NLPS^H = 12\ dB$ $D_P = 6.4\frac{AB}{\lambda^2}$: $\eta_{il} = 0.51$

Bocina plano E	
Diferencia de fases	$K\delta = K\frac{y^2}{2L_E}$
Campo iluminante	$\vec{E} = \hat{j}E_0\cos\left(\frac{\pi}{a}x\right)e^{-j\beta\frac{y^2}{2L_E}}$
Campo	$E_\theta = j\frac{e^{-jkr}}{2\lambda r}\left(1+\frac{\eta}{Z_0}\cos\theta\right)\cos\phi\int_{-B/2}^{B/2}\int_{-a/2}^{a/2}E_0\cos\left(\frac{\pi}{a}x'\right)e^{-j\beta\frac{y'^2}{2L_E}}e^{jk_x x'}e^{jk_y y'}dx'dy'$
	$E_\phi = j\frac{e^{-jkr}}{2\lambda r}\left(\frac{\eta}{Z_0}+\cos\theta\right)sen\phi\int_{-B/2}^{B/2}\int_{-a/2}^{a/2}E_0\cos\left(\frac{\pi}{a}x'\right)e^{-j\beta\frac{y'^2}{2L_E}}e^{jk_x x'}e^{jk_y y'}dx'dy'$
Densidad de Potencia	$\wp_{max} = \frac{8E_0^2a^2L_E}{\eta\pi^2\lambda r^2}\left[C^2\left(\frac{B}{\sqrt{2\lambda L_E}}\right)+S^2\left(\frac{B}{\sqrt{2\lambda L_E}}\right)\right]$
Direct.	$D_E = \frac{64aL_E}{\pi\lambda B}\left[C^2\left(\frac{B}{\sqrt{2\lambda L_E}}\right)+S^2\left(\frac{B}{\sqrt{2\lambda L_E}}\right)\right]$
Parámetros óptimos	$B_{opt} = \sqrt{2\lambda L_E}$ $\Delta\theta^E_{-3dB} = 56\frac{\lambda}{B}(grados)$; $\Delta\theta^H_{-3dB} = 67\frac{\lambda}{a}(grados)$ $NLPS^E = 10dB$; $NLPS^H = 23dB$ $D_E = 8\frac{aB}{\lambda^2}$: $\eta_{il} = 0.64$

Reflectores parabólicos

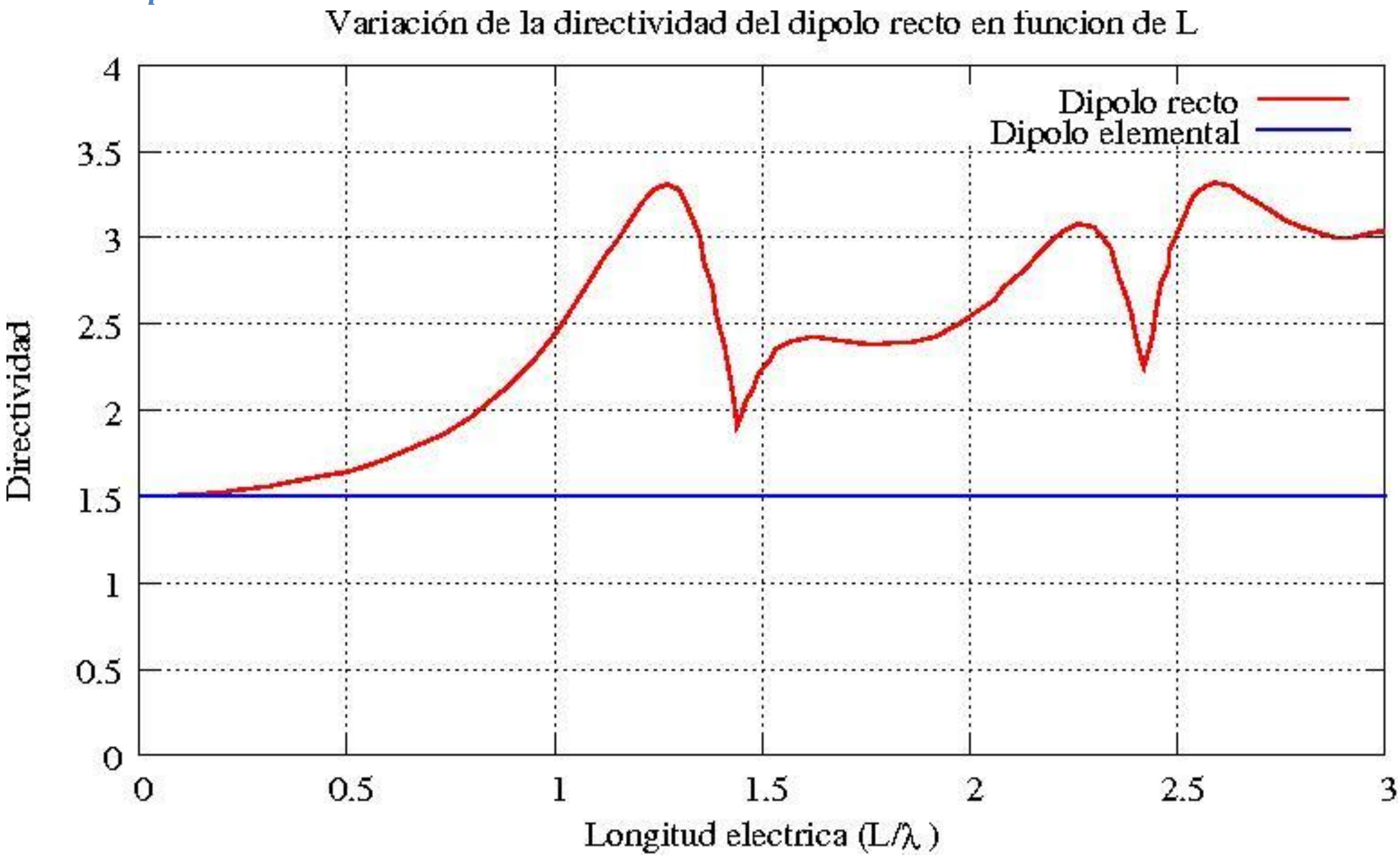
Ecuaciones geométricas de la parábola	Ecuación polar de la parábola	$r(\theta)=\frac{2f}{1+\cos(\theta)}=\frac{f}{\cos^2\left(\frac{\theta}{2}\right)}$	Ecuación cartesiana de la parábola	$y^2=4f(f-z)$
	Distancia focal sobre diámetro	$\frac{f}{D}=\frac{1}{4\tan\left(\frac{\beta}{2}\right)}=\frac{1}{4}\cot\left(\frac{\beta}{2}\right)$	Teorema de Malus-Dupin	$r+z=2f=cte$
Ecuaciones electromagnéticas de la parábola	Campo en la apertura	$\vec{E}_a=\sqrt{\frac{\eta P_r G_f(\theta,\phi)}{4\pi}}\frac{e^{-2jkf}}{r}\hat{e}_r$	Campos	$E_\theta=j\frac{e^{-jkr}}{2\lambda r}(1+\cos\theta)\iint_{S'}\left[E_x^a\cos\phi+E_y^a sen\phi\right]e^{j\vec{k}\cdot\vec{r}'}dS'$
	Ley de la iluminación	$\tau=20\log\left(\cos^2\left(\frac{\theta}{2}\right)\right)+20\log\sqrt{G(\theta,\phi)}$		$E_\phi=j\frac{e^{-jkr}}{2\lambda r}(1+\cos\theta)\iint_{S'}\left[E_x^a sen\phi+E_y^a\cos\phi\right]e^{j\vec{k}\cdot\vec{r}'}dS'$
Eficiencias y pérdidas	Eficiencia de bloqueo	$\eta_{bl}=20\log\left(\frac{G}{G_0}\right)\approx 20\log\left(1-2\left(\frac{d_s}{D}\right)^2\right)$	Eficiencia de apertura	$\eta_{il}=32\left(\frac{f}{D}\right)^2\frac{\left\lvert\int_0^\beta\sqrt{G(\theta)}\tan\left(\frac{\theta}{2}\right)d\theta\right\rvert^2}{\int_0^\beta G(\theta)sen(\theta)d\theta}$
	Eficiencia de desbordamiento	$\eta_s=\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\beta}G_f(\theta,\phi)sen\theta d\theta d\phi$	Pérdidas por desplazamiento axial	$G_A(dB)=20\log\left(\frac{sen(X)}{X}\right)$ con $X=\frac{2\pi d_z/\lambda}{1+\left(\frac{4f}{D}\right)^2}$
	Pérdida por desplazamiento lateral	$\theta_d=BDF\psi_d$ con $\psi_d=\tan^{-1}\left(\frac{d_1}{F}\right)$	Ganancia	$G=\frac{4\pi}{\lambda^2}S_a\eta$ Refl. de apertura circular $G=\left(\frac{\pi D}{\lambda}\right)^2\prod_i\eta_i$

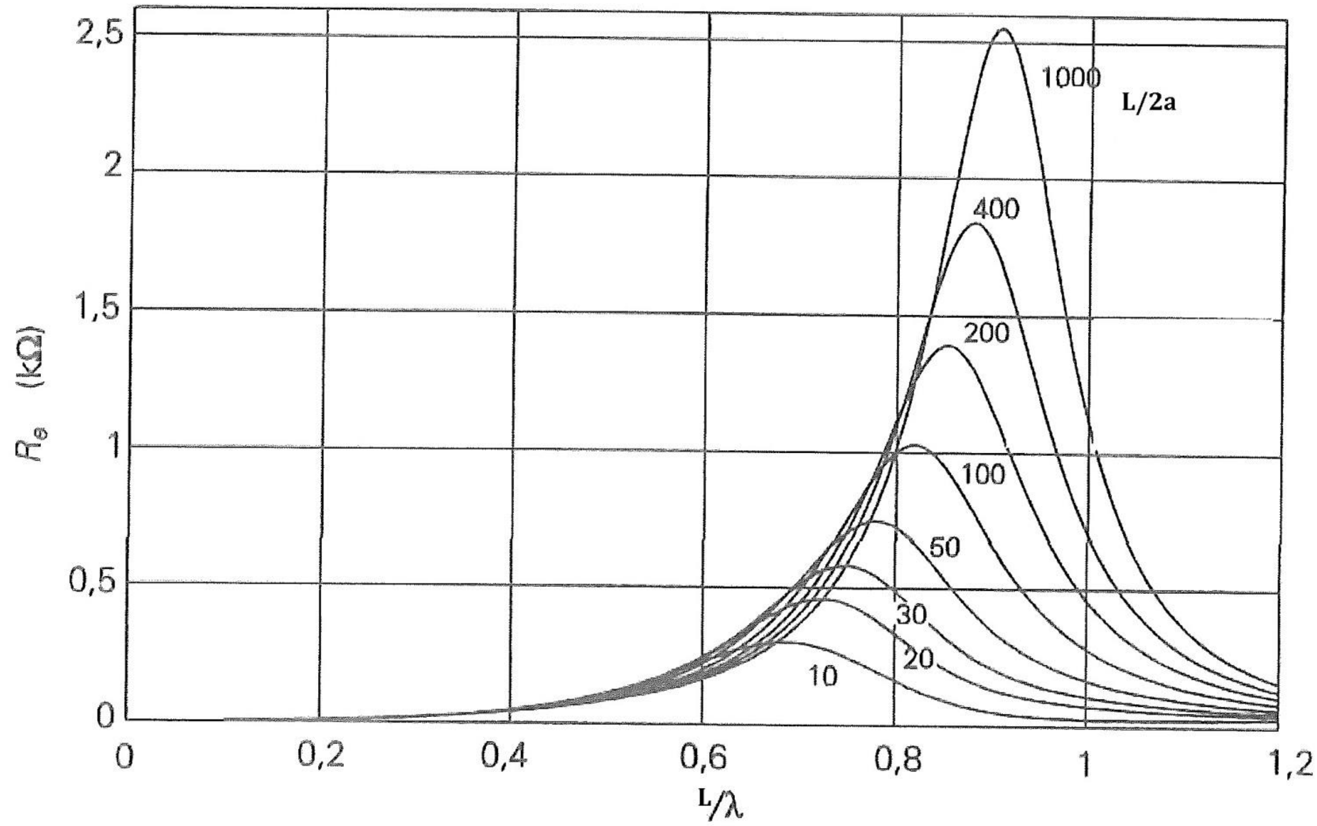
Propagación en entorno natural

Propagación	Fórmula de Friis	$l_B=\frac{P_{rec}}{P_{trans}}=\left(1-\lvert\Gamma_T\rvert^2\right)\left(1-\lvert\Gamma_R\rvert^2\right)G_TG_R\lvert\hat{e}_T\bullet\hat{e}_R\rvert^2\left(\frac{\lambda}{4\pi d}\right)^2$			
		$L_B=L_0+10\log\left(1-\lvert\Gamma_T\rvert^2\right)+10\log\left(1-\lvert\Gamma_R\rvert^2\right)+10\log G_T+10\log G_R+20\log\lvert\hat{e}_T\bullet\hat{e}_R\rvert[dB]$			
	Ecuación del Radar	Biestático	$\frac{P_{rec}}{P_{trans}}=\left(1-\lvert\Gamma_T\rvert^2\right)\left(1-\lvert\Gamma_R\rvert^2\right)G_TG_R\lvert\hat{e}_T\bullet\hat{e}_R\rvert^2\left(\frac{\lambda}{4\pi R_1R_2}\right)^2\frac{\sigma}{4\pi}$		
		Monoestático con adaptación perfecta	$\frac{P_{rec}}{P_{trans}}=\frac{G^2\sigma\lambda^2}{(4\pi)^3R^4}$		
Reflexión en tierra plana	Coeficientes de reflexión	$\Gamma_\perp=\frac{n_1\cos\theta_i-n_2\cos\theta_t}{n_1\cos\theta_i+n_2\cos\theta_t}$ $\Gamma_\parallel=\frac{n_2\cos\theta_t-n_1\cos\theta_i}{n_2\cos\theta_t+n_1\cos\theta_i}$	Coeficientes de transmisión	$\tau_\perp=\frac{E_{t0}}{E_{i0}}=\frac{2n_1\cos\theta_i}{n_1\cos\theta_i+n_2\cos\theta_t}$ $\tau_\parallel=\frac{2n_2\cos\theta_i}{n_2\cos\theta_t+n_1\cos\theta_i}$	
	Índice de refracción	$n_2=\sqrt{\varepsilon'-j\frac{\sigma}{\omega\varepsilon_0}}$	Friis considerando tierra plana	$\frac{P_R}{P_T}=\frac{1}{4\pi R^2}D_TA_{efR}\left(\frac{2kh_1h_2}{R}\right)^2$	
Difracción	1ª región de Fresnel	$R_1=\sqrt{\lambda\frac{d_1d_2}{d_1+d_2}}$	Esfericidad de la Tierra	$h=\frac{d_1d_2}{2kR_t}>5$	
	Pérdidas o Ganancia por Tierra esférica	$20\log\left(\frac{E}{E_0}\right)=F(d)+H(h_1)+H(h_2)$			
Onda de Superficie	Condiciones de validez	$h\le 12\sigma^{\frac{1}{2}}\lambda^{\frac{3}{2}}\ (m)$	$\varepsilon_r\ll 60\lambda\sigma$	Relación entre campos	$\lvert E\rvert=\lvert E\rvert_{ref}\sqrt{\frac{PIRE}{PIRE_{ref}}}$
Troposfera	Atenuación específica por lluvia	$\gamma_R=KR^\alpha\quad[dB/km]$	Refracción	Refractividad	$N=77.6\frac{P}{T}+3.73\cdot10^5\frac{e}{T^2}$
				Refractividad en atmósfera de referencia	$N(h)=315e^{-0.136h}$
				Radio de curvatura	$\frac{1}{r}=\frac{dn}{dh}=10^{-6}\frac{dN}{dh}$
Ionosfera	Frecuencia del plasma	$f_p=9\sqrt{N}\quad[\mathrm{Hz}]$	Rotación de Faraday	$tg\Phi=tg\left[(k_2-k_1)\frac{l}{2}\right]$	
	MUF	$MUF=\frac{f_p}{\cos\varphi_i}=\frac{f_p}{sen\Psi}$	Distancia mínima cubierta por reflexión ionosférica	$d_{mín}=2h\sqrt{\left(\frac{f}{f_p}\right)^2-1}$	

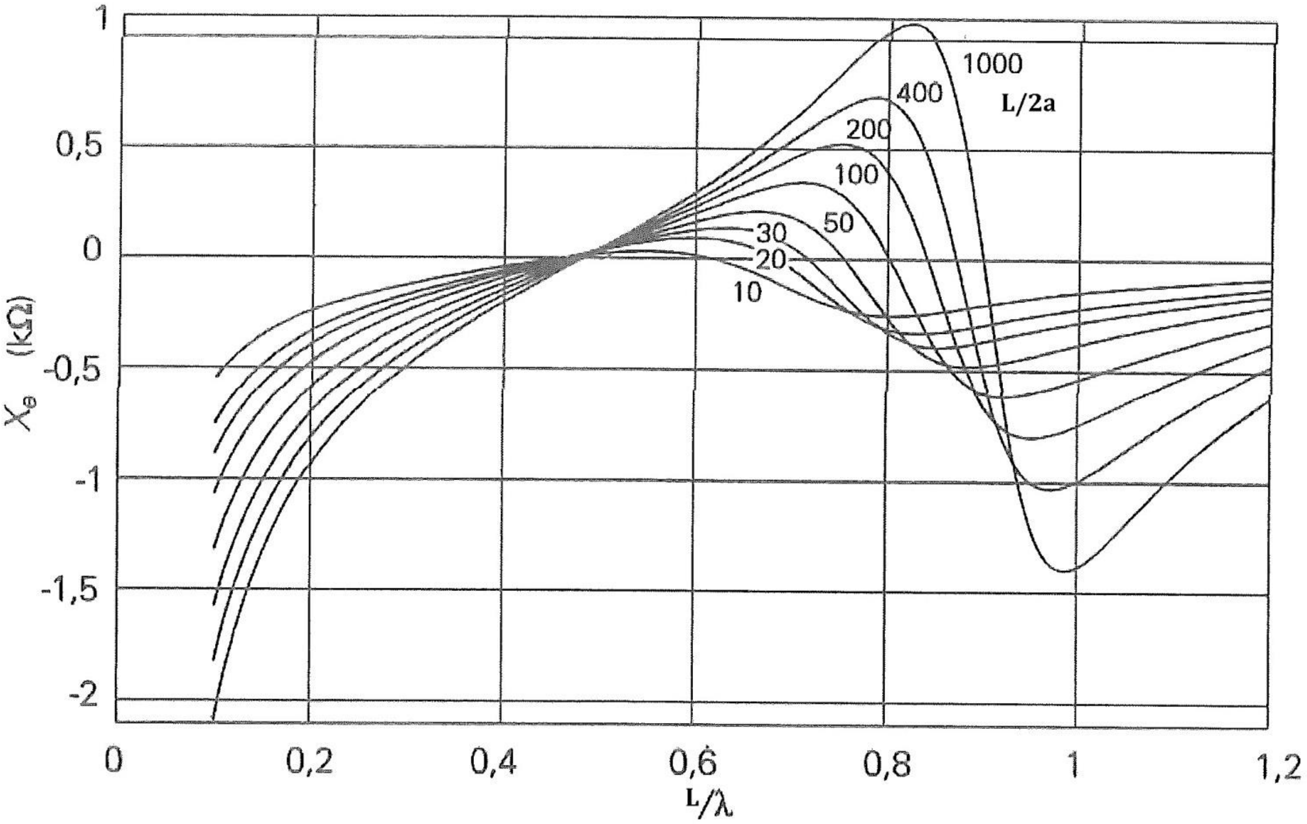
Gráficas del tema de antenas lineales

Antenas Dipolo



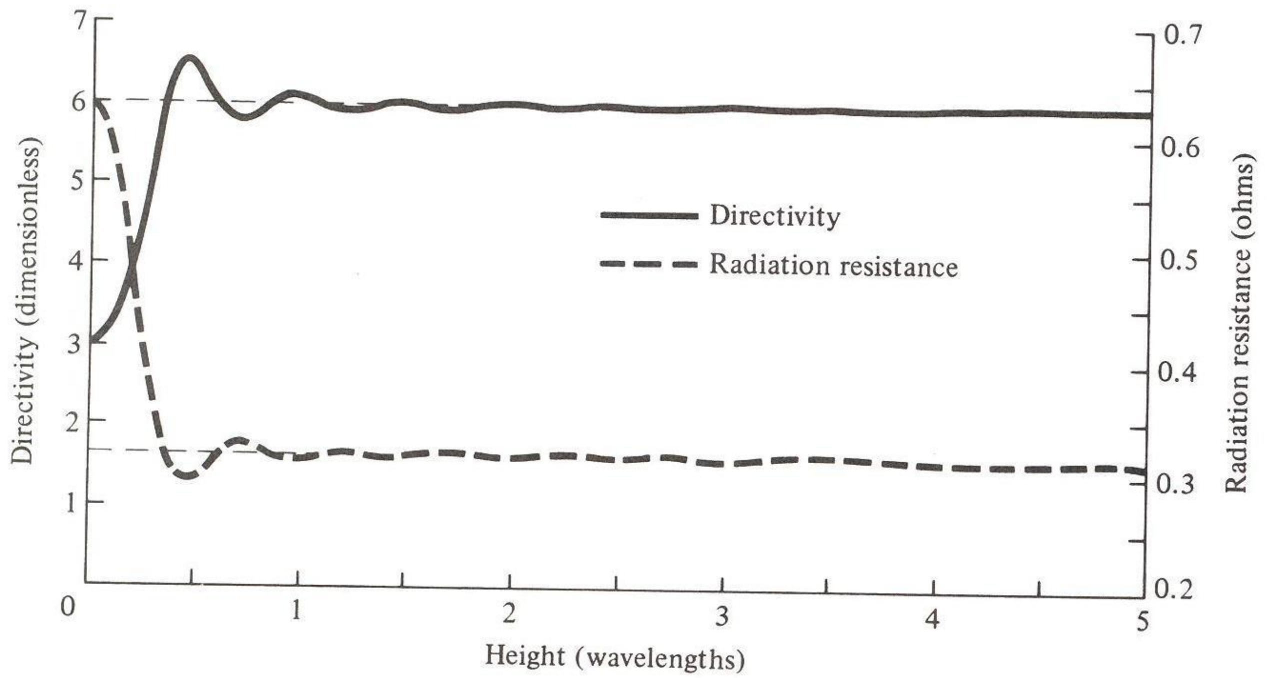


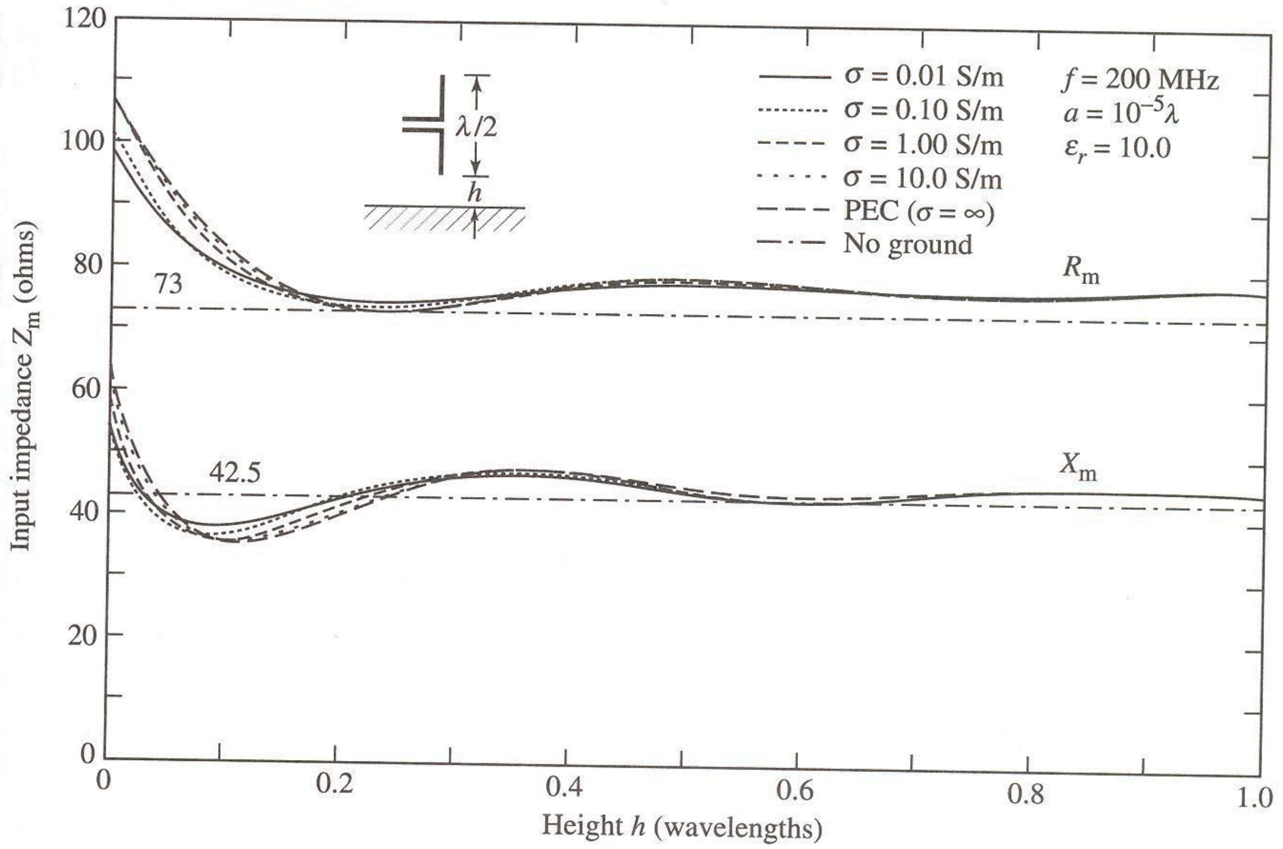
2,5
2
1,5
1
0,5
0
R_e (kΩ)
1000
L/2a
400
200
100
50
30
20
10
0
0,2
0,4
0,6
0,8
1
1,2
L/λ



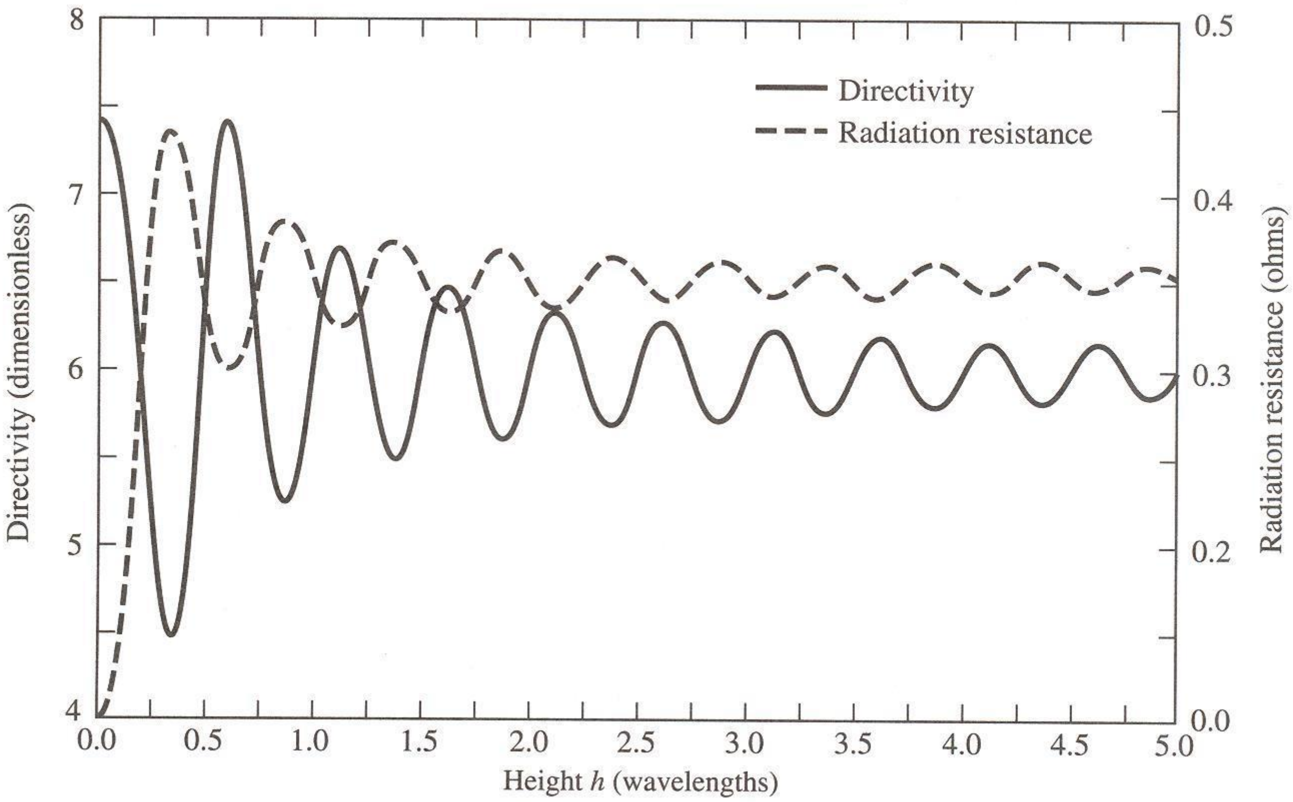
1
0,5
0
-0,5
-1
-1,5
-2
X_e (kΩ)
1000
400
L/2a
200
100
50
30
20
10
0
0,2
0,4
0,6
0,8
1
1,2
L/λ

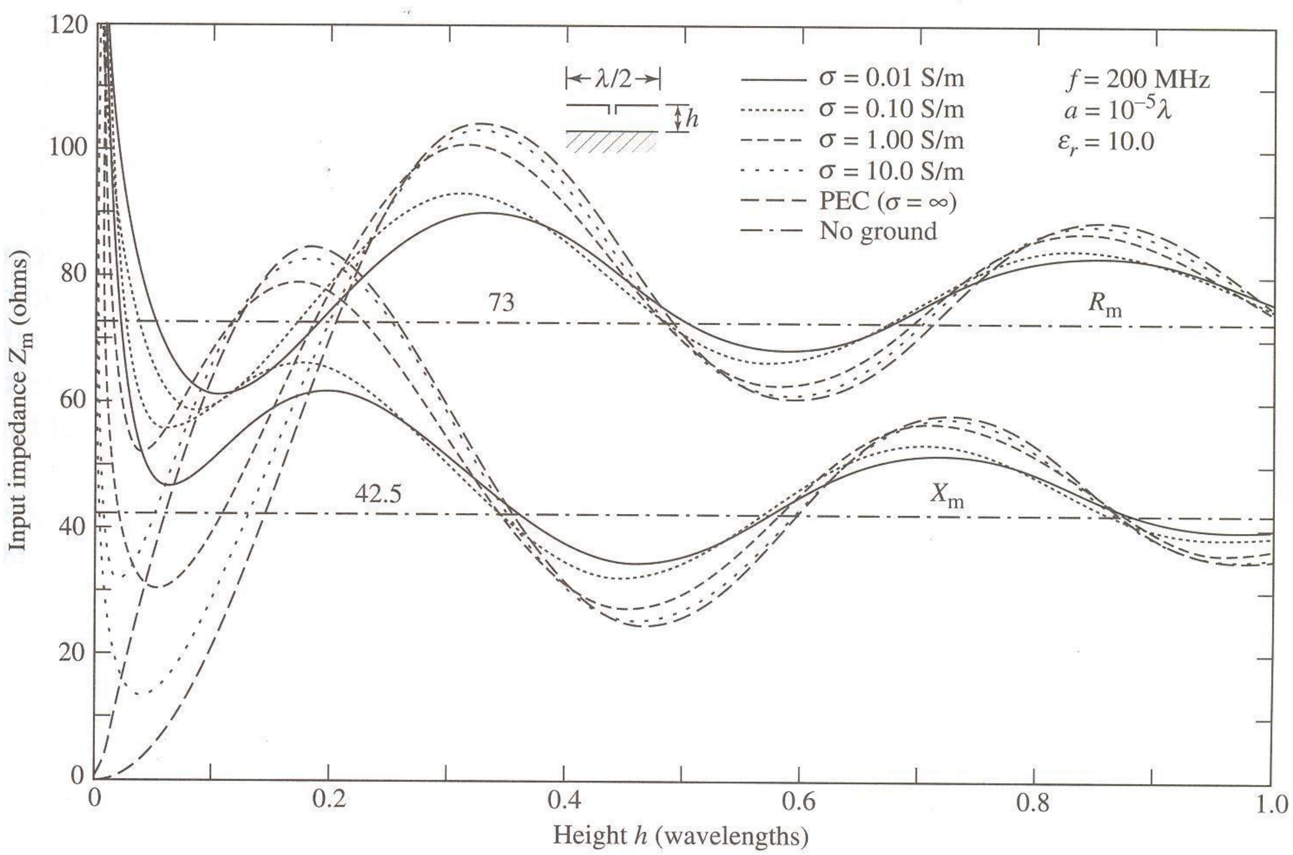
Dipolo Eléctrico Vertical frente a Plano a Tierra



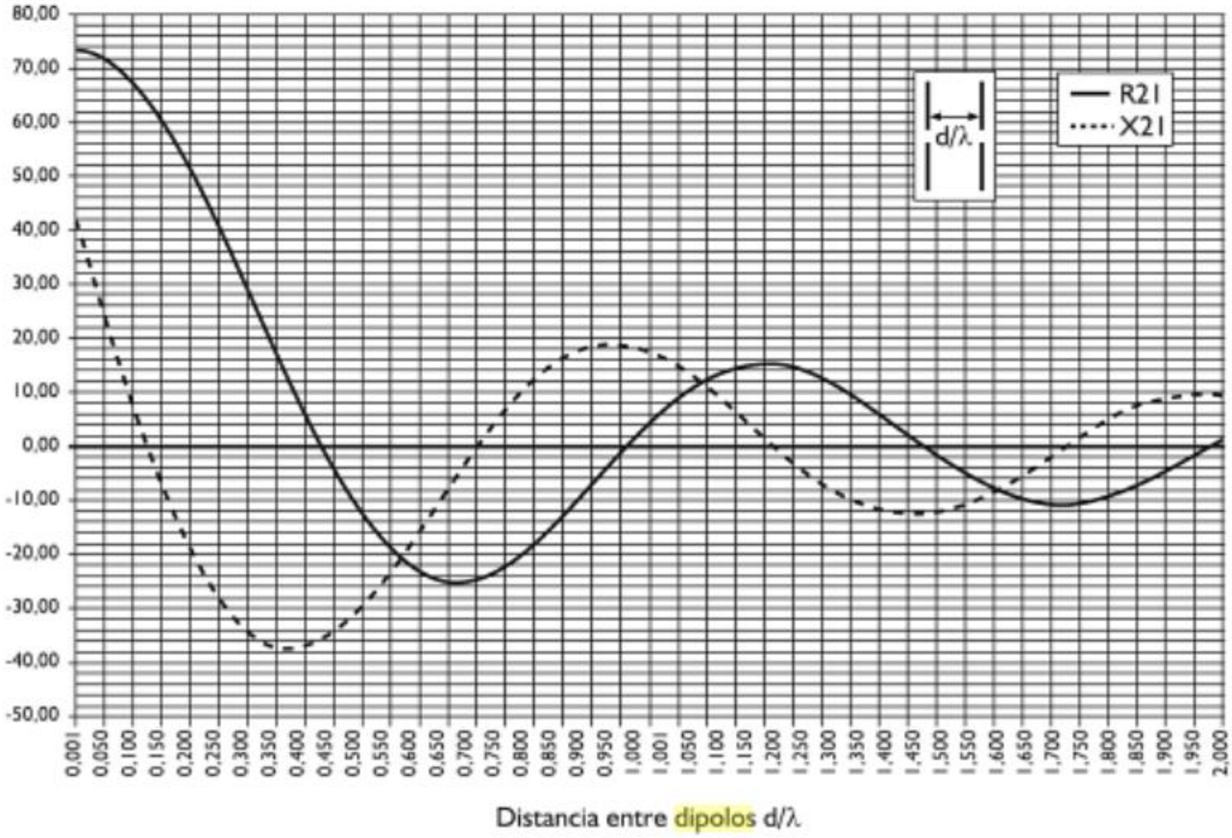


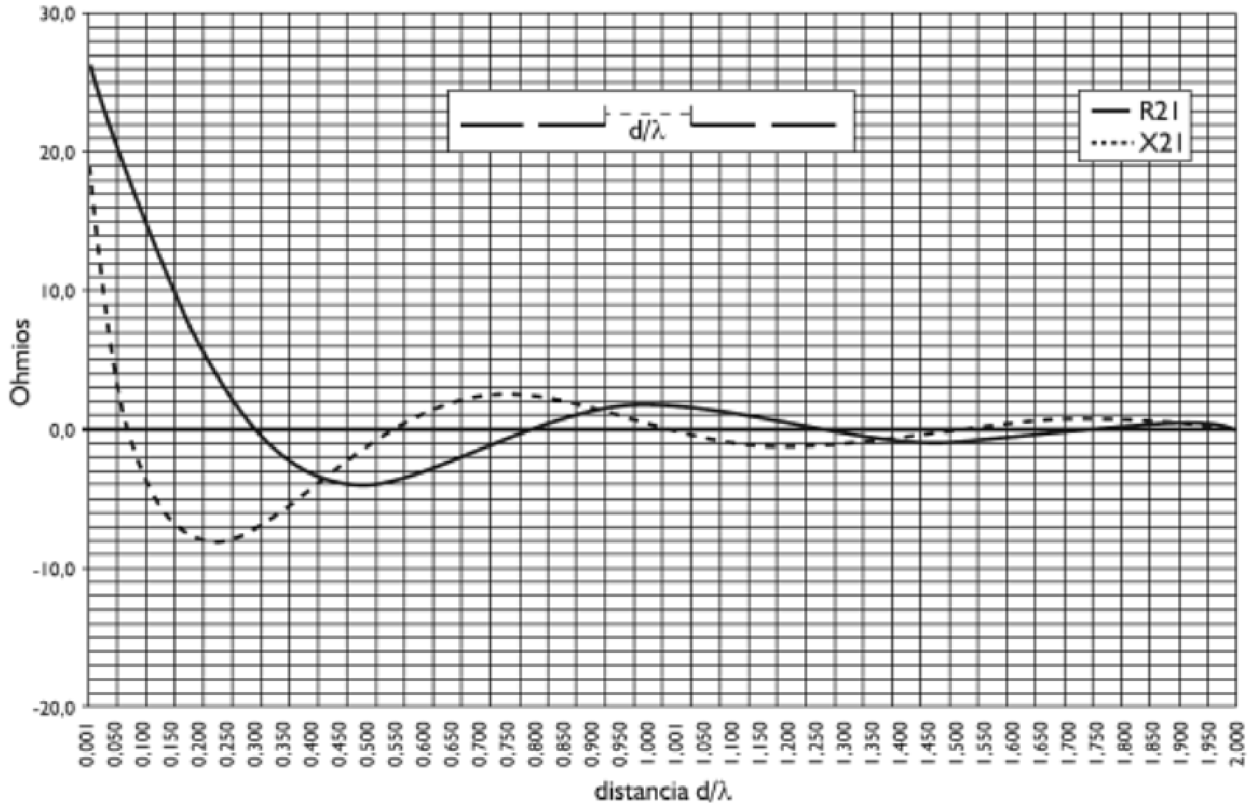
Dipolo Eléctrico Horizontal frente a un plano a tierra



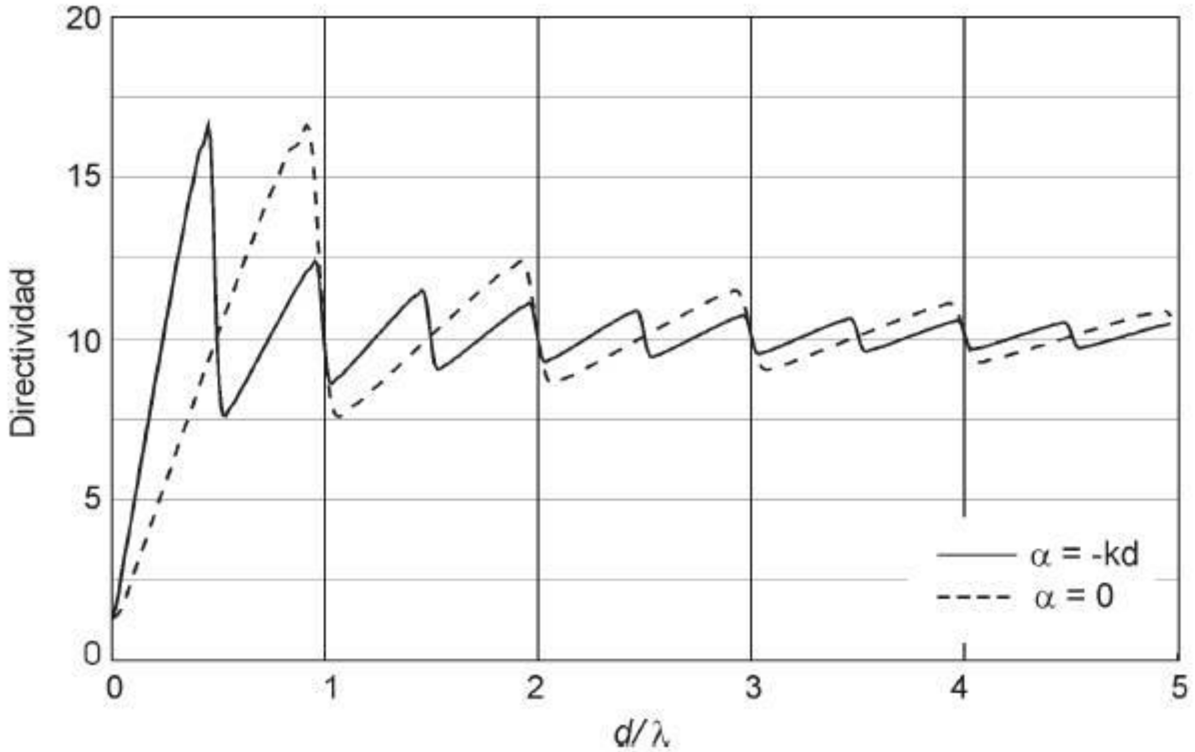


Impedancia mutua entre dipolos



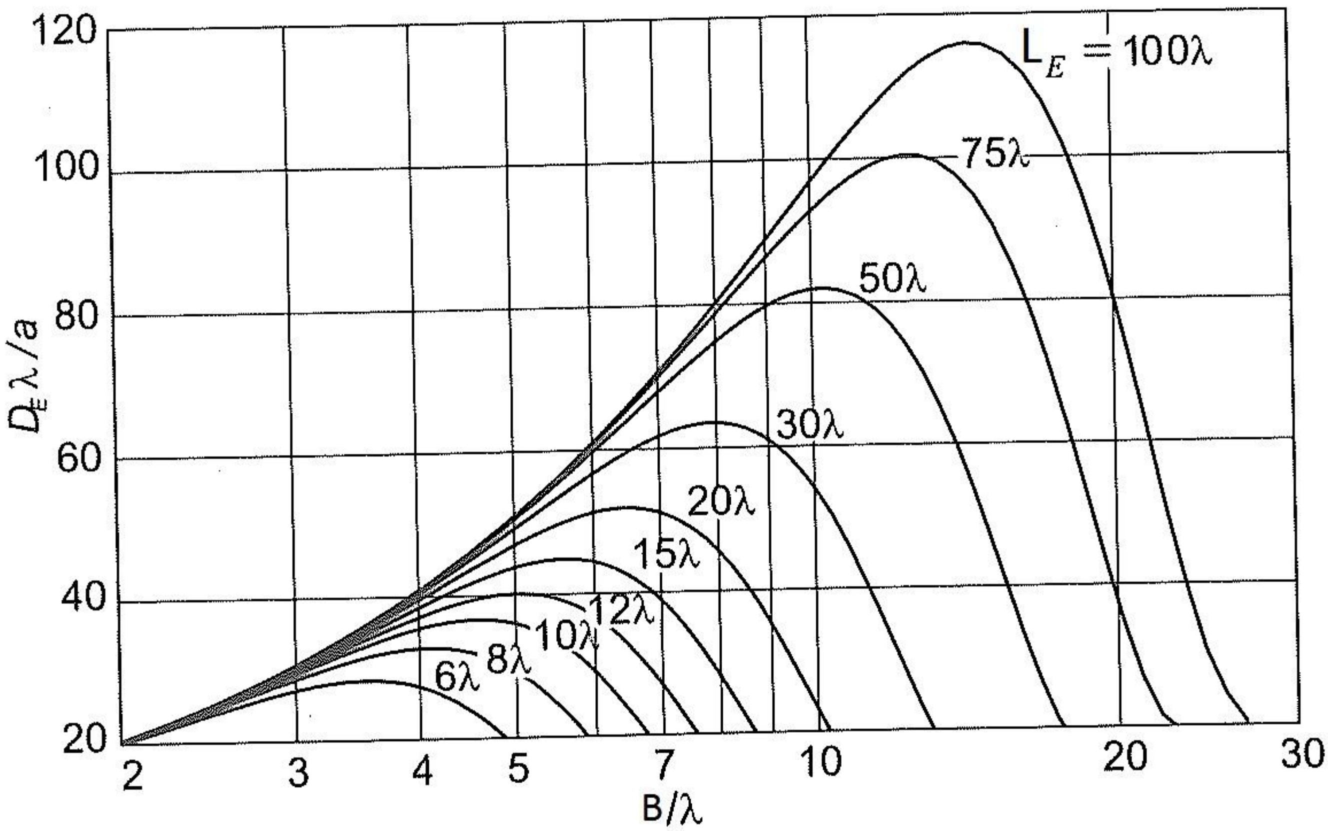


Gráficas del tema de arrays de antenas lineales

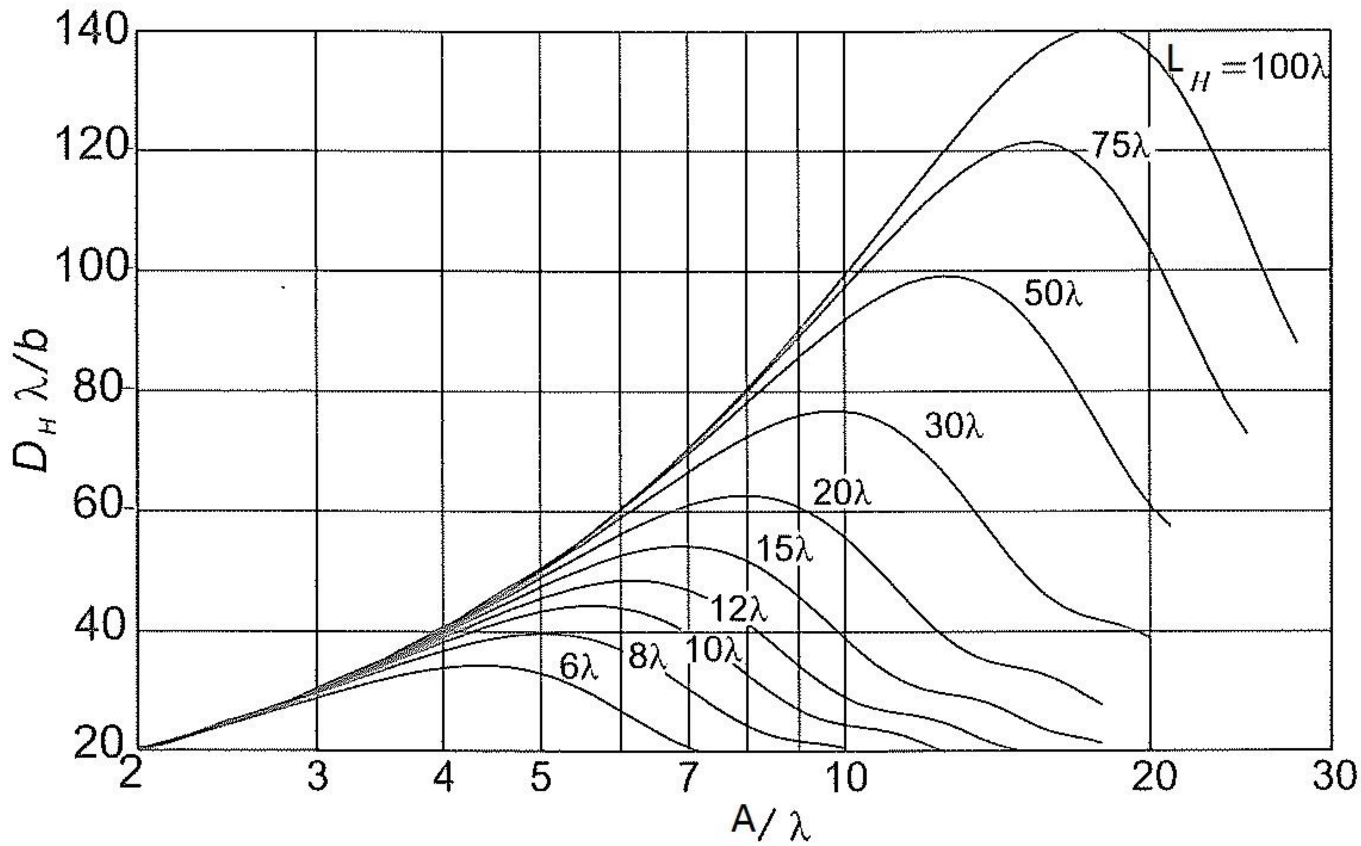


Gráficas del tema de antenas de apertura, bocinas y reflectores

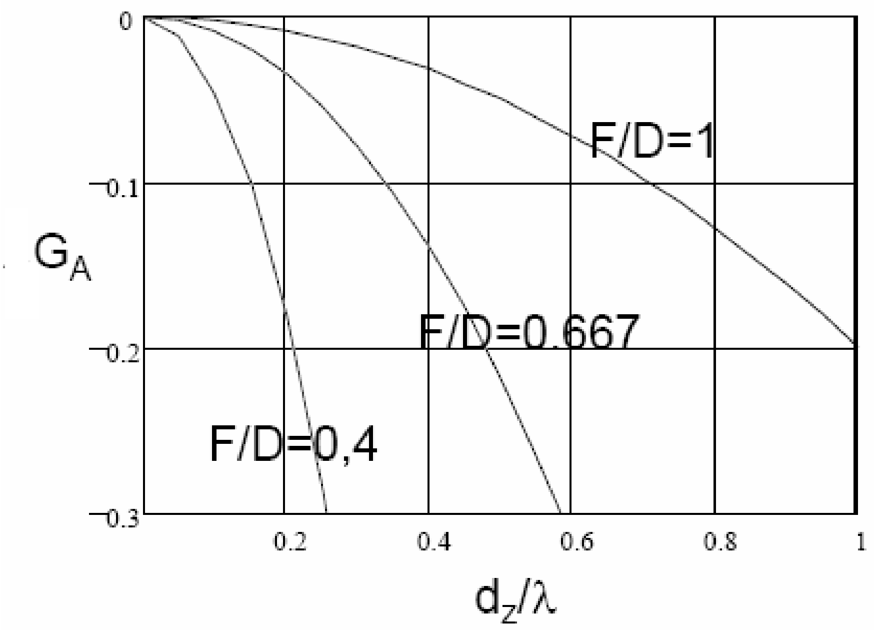
Directividad normalizada de una Bocina en Plano-E



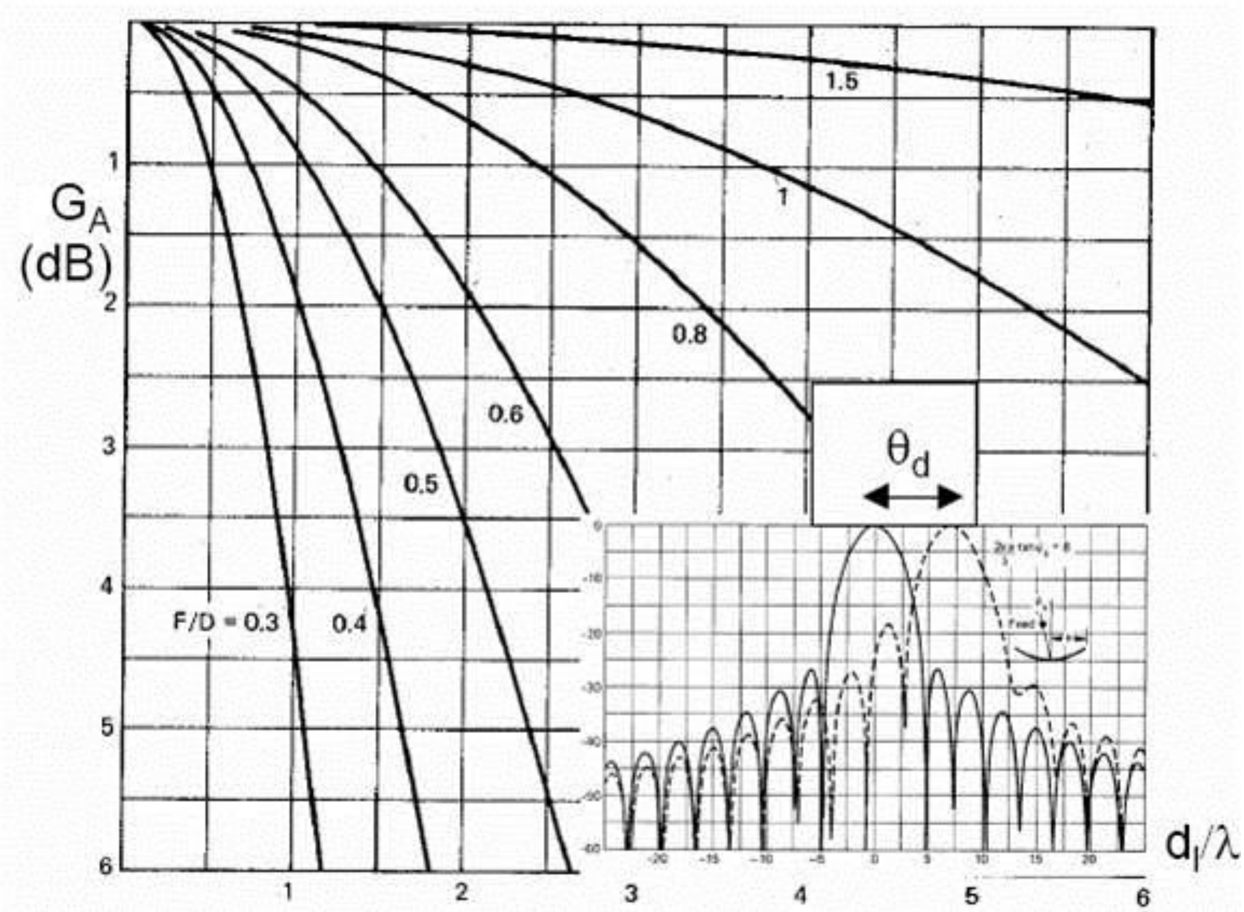
Directividad normalizada de una Bocina en Plano H



Pérdidas por desplazamiento axial



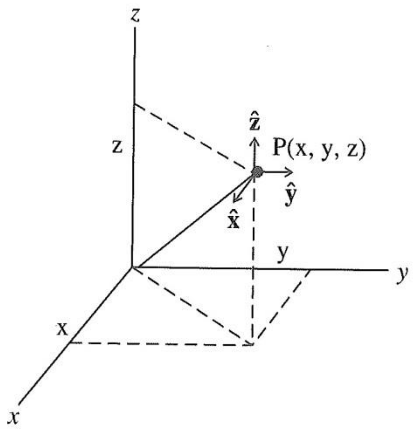
Pérdidas por desplazamiento lateral



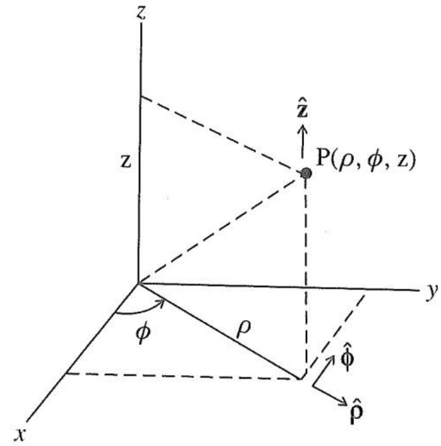
Sistemas de Coordenadas y Vectores

C.1 THE COORDINATE SYSTEMS AND UNIT VECTORS

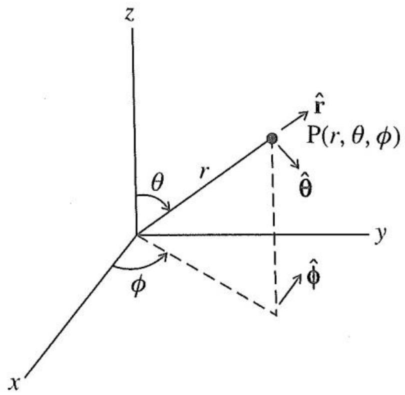
C.1.1 The Coordinate Systems



Rectangular Coordinates



Cylindrical Coordinates



Spherical Coordinates

C.1.2 Unit Vector Representations

$$\hat{\mathbf{x}} = \hat{\mathbf{r}} \sin\theta \cos\phi + \hat{\boldsymbol{\theta}} \cos\theta \cos\phi - \hat{\boldsymbol{\phi}} \sin\phi$$

$$\hat{\mathbf{y}} = \hat{\mathbf{r}} \sin\theta \sin\phi + \hat{\boldsymbol{\theta}} \cos\theta \cos\phi + \hat{\boldsymbol{\phi}} \cos\phi$$

$$\hat{\mathbf{z}} = \hat{\mathbf{r}} \cos\theta - \hat{\boldsymbol{\theta}} \sin\theta$$

$$\hat{\mathbf{r}} = \hat{\mathbf{x}} \sin\theta \cos\phi + \hat{\mathbf{y}} \sin\theta \sin\phi + \hat{\mathbf{z}} \cos\theta$$

$$\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}} \cos\theta \cos\phi + \hat{\mathbf{y}} \cos\theta \sin\phi - \hat{\mathbf{z}} \sin\theta$$

$$\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin\phi + \hat{\mathbf{y}} \cos\phi$$

C.2 VECTOR IDENTITIES

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$$

$$(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = (\mathbf{C} \cdot \mathbf{A})\mathbf{B} - (\mathbf{C} \cdot \mathbf{B})\mathbf{A}$$

$$\nabla \cdot (\nabla \times \mathbf{G}) = 0$$

$$\nabla \times \nabla g = 0$$

$$\nabla \cdot \nabla g = \nabla^2 g$$

$$\nabla(f + g) = \nabla f + \nabla g$$

$$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$$

$$\nabla(fg) = g\nabla f + f\nabla g$$

$$\nabla \cdot (f\mathbf{G}) = \mathbf{G} \cdot (\nabla f) + f(\nabla \cdot \mathbf{G})$$

$$\nabla \times (f\mathbf{G}) = (\nabla f) \times \mathbf{G} + f(\nabla \times \mathbf{G})$$

$$\nabla \times (\nabla \times \mathbf{G}) = \nabla(\nabla \cdot \mathbf{G}) - \nabla^2 \mathbf{G}$$

$$\nabla^2 \mathbf{G} = \hat{\mathbf{x}}\nabla^2 G_x + \hat{\mathbf{y}}\nabla^2 G_y + \hat{\mathbf{z}}\nabla^2 G_z$$

$$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot (\nabla \times \mathbf{F}) - \mathbf{F} \cdot (\nabla \times \mathbf{G})$$

$$\mathbf{F} \cdot (\mathbf{G} \times \mathbf{H}) = \mathbf{G} \cdot (\mathbf{H} \times \mathbf{F}) = \mathbf{H} \cdot (\mathbf{F} \times \mathbf{G})$$

$$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$$

$$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$$

$$\iiint_v \nabla \cdot \mathbf{G}\, dv = \oiint_s \mathbf{G} \cdot d\mathbf{s} \quad \text{divergence theorem}$$

$$\iiint_s (\nabla \times \mathbf{G}) \cdot d\mathbf{s} = \oint_l \mathbf{G} \cdot d\mathbf{l} \quad \text{Stokes' theorem}$$

C.3 VECTOR DIFFERENTIAL OPERATORS

C.3.1 Rectangular Coordinates

$$\nabla g = \hat{\mathbf{x}}\frac{\partial g}{\partial x} + \hat{\mathbf{y}}\frac{\partial g}{\partial y} + \hat{\mathbf{z}}\frac{\partial g}{\partial z}$$

$$\nabla \cdot \mathbf{G} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y} + \frac{\partial G_z}{\partial z}$$

$$\nabla \times \mathbf{G} = \hat{\mathbf{x}}\left(\frac{\partial G_z}{\partial y} - \frac{\partial G_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial G_x}{\partial z} - \frac{\partial G_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial G_y}{\partial x} - \frac{\partial G_x}{\partial y}\right)$$

$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2}$$

C.3.2 Cylindrical Coordinates

$$\nabla g = \hat{\boldsymbol{\rho}}\frac{\partial g}{\partial \rho} + \hat{\boldsymbol{\phi}}\frac{1}{\rho}\frac{\partial g}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial g}{\partial z}$$

$$\nabla \cdot \mathbf{G} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho G_\rho) + \frac{1}{\rho}\frac{\partial G_\phi}{\partial \phi} + \frac{\partial G_z}{\partial z}$$

$$\nabla \times \mathbf{G} = \hat{\boldsymbol{\rho}}\left(\frac{1}{\rho}\frac{\partial G_z}{\partial \phi} - \frac{\partial G_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial G_\rho}{\partial z} - \frac{\partial G_z}{\partial \rho}\right) + \hat{\mathbf{z}}\frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho G_\phi) - \frac{\partial G_\rho}{\partial \phi}\right]$$

$$\nabla^2 g = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial g}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 g}{\partial \phi^2} + \frac{\partial^2 g}{\partial z^2}$$

Polinomios de Chebyshev

$T_0(x)=1$
$T_1(x)=x$
$T_2(x) = 2x^2 - 1$
$T_3(x) = 4x^3 - 3x$
$T_4(x) = 8x^4 - 8x^2 + 1$
$T_5(x) = 16x^5 - 20x^3 + 5x$
$T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$

Los polinomios de Chebyshev se obtienen mediante la siguiente relación de recurrencia:

$T_0(x)=1$
$T_1(x)=x$
$T_n(x)=2x \cdot T_{n-1}(x)-T_{n-2}(x)$

Tablas

Función Sinc(x)

$$f(x) = \frac{\sin(x)}{x}$$

x	$\sin(x)/x$	x	$\sin(x)/x$	x	$\sin(x)/x$
0.0	1.00000	2.8	0.11964	5.6	−0.11273
0.1	0.99833	2.9	0.08250	5.7	−0.09661
0.2	0.99335	3.0	0.04704	5.8	−0.08010
0.3	0.98507	3.1	0.01341	5.9	−0.06337
0.4	0.97355	3.2	−0.01824	6.0	−0.04657
0.5	0.95885	3.3	−0.04780	6.1	−0.02986
0.6	0.94107	3.4	−0.07516	6.2	−0.01340
0.7	0.92031	3.5	−0.10022	6.3	0.00267
0.8	0.89670	3.6	−0.12292	6.4	0.01821
0.9	0.87036	3.7	−0.14320	6.5	0.03309
1.0	0.84147	3.8	−0.16101	6.6	0.04720
1.1	0.81019	3.9	−0.17635	6.7	0.06042
1.2	0.77670	4.0	−0.18920	6.8	0.07266
1.3	0.74120	4.1	−0.19958	6.9	0.08383
1.4	0.70389	4.2	−0.20752	7.0	0.09385
1.5	0.66500	4.3	−0.21306	7.1	0.10267
1.6	0.62473	4.4	−0.21627	7.2	0.11023
1.7	0.58333	4.5	−0.21723	7.3	0.11650
1.8	0.54103	4.6	−0.21602	7.4	0.12145
1.9	0.49805	4.7	−0.21275	7.5	0.12507
2.0	0.45465	4.8	−0.20753	7.6	0.12736
2.1	0.41105	4.9	−0.20050	7.7	0.12833
2.2	0.36750	5.0	−0.19179	7.8	0.12802
2.3	0.32422	5.1	−0.18153	7.9	0.12645
2.4	0.28144	5.2	−0.16990	8.0	0.12367
2.5	0.23939	5.3	−0.15703	8.1	0.11974
2.6	0.19827	5.4	−0.14310	8.2	0.11472
2.7	0.15829	5.5	−0.12828	8.3	0.10870

x	$\sin(x)/x$	x	$\sin(x)/x$	x	$\sin(x)/x$
8.4	0.10174	10.7	−0.08941	13.0	0.03232
8.5	0.09394	10.8	−0.09083	13.1	0.03883
8.6	0.08540	10.9	−0.09132	13.2	0.04485
8.7	0.07620	11.0	−0.09091	13.3	0.05034
8.8	0.06647	11.1	−0.08960	13.4	0.05525
8.9	0.05629	11.2	−0.08743	13.5	0.05954
9.0	0.04579	11.3	−0.08443	13.6	0.06317
9.1	0.03507	11.4	−0.08064	13.7	0.06613
9.2	0.02423	11.5	−0.07613	13.8	0.06838
9.3	0.01338	11.6	−0.07093	13.9	0.06993
9.4	0.00264	11.7	−0.06513	14.0	0.07076
9.5	−0.00791	11.8	−0.05877	14.1	0.07087
9.6	−0.01816	11.9	−0.05194	14.2	0.07028
9.7	−0.02802	12.0	−0.04471	14.3	0.06901
9.8	−0.03740	12.1	−0.03716	14.4	0.06706
9.9	−0.04622	12.2	−0.02936	14.5	0.06448
10.0	−0.05440	12.3	−0.02140	14.6	0.06129
10.1	−0.06189	12.4	−0.01336	14.7	0.05753
10.2	−0.06861	12.5	−0.00531	14.8	0.05326
10.3	−0.07453	12.6	0.00267	14.9	0.04852
10.4	−0.07960	12.7	0.01049	15.0	0.04335
10.5	−0.08378	12.8	0.01809		
10.6	−0.08705	12.9	0.02539		

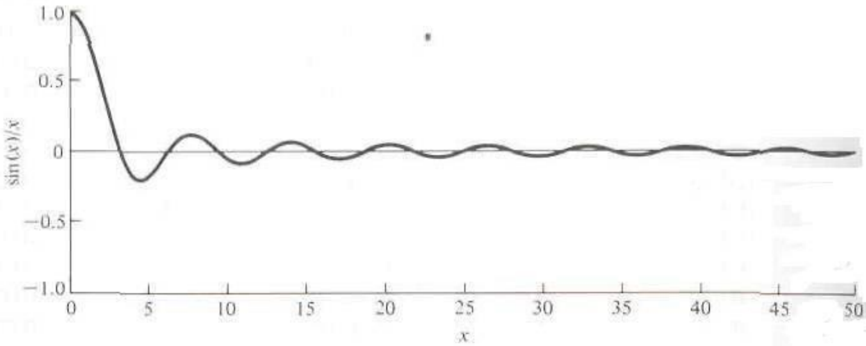


Figure I.1 Plot of sin $(x)/x$ function.

Funciones de Bessel

Funciones de Bessel de primer tipo

beta	J0	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10	J0/J1
0.0	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	Inf
0.1	0.998	0.050	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	#####
0.2	0.990	0.100	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	9.950
0.3	0.978	0.148	0.011	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	6.591
0.4	0.960	0.196	0.020	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	4.899
0.5	0.938	0.242	0.031	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.874
0.6	0.912	0.287	0.044	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.181
0.7	0.881	0.329	0.059	0.007	0.001	0.000	0.000	0.000	0.000	0.000	0.000	2.678
0.8	0.846	0.369	0.076	0.010	0.001	0.000	0.000	0.000	0.000	0.000	0.000	2.294
0.9	0.808	0.406	0.095	0.014	0.002	0.000	0.000	0.000	0.000	0.000	0.000	1.989
1.0	0.765	0.440	0.115	0.020	0.002	0.000	0.000	0.000	0.000	0.000	0.000	1.739
1.1	0.720	0.471	0.137	0.026	0.004	0.000	0.000	0.000	0.000	0.000	0.000	1.528
1.2	0.671	0.498	0.159	0.033	0.005	0.001	0.000	0.000	0.000	0.000	0.000	1.347
1.3	0.620	0.522	0.183	0.041	0.007	0.001	0.000	0.000	0.000	0.000	0.000	1.188
1.4	0.567	0.542	0.207	0.050	0.009	0.001	0.000	0.000	0.000	0.000	0.000	1.046
1.5	0.512	0.558	0.232	0.061	0.012	0.002	0.000	0.000	0.000	0.000	0.000	0.917
1.6	0.455	0.570	0.257	0.073	0.015	0.002	0.000	0.000	0.000	0.000	0.000	0.799
1.7	0.398	0.578	0.282	0.085	0.019	0.003	0.000	0.000	0.000	0.000	0.000	0.689
1.8	0.340	0.582	0.306	0.099	0.023	0.004	0.001	0.000	0.000	0.000	0.000	0.585
1.9	0.282	0.581	0.330	0.113	0.028	0.006	0.001	0.000	0.000	0.000	0.000	0.485
2.0	0.224	0.577	0.353	0.129	0.034	0.007	0.001	0.000	0.000	0.000	0.000	0.388
2.1	0.167	0.568	0.375	0.145	0.040	0.009	0.002	0.000	0.000	0.000	0.000	0.293
2.2	0.110	0.556	0.395	0.162	0.048	0.011	0.002	0.000	0.000	0.000	0.000	0.199
2.3	0.056	0.540	0.414	0.180	0.056	0.013	0.003	0.000	0.000	0.000	0.000	0.103
2.4	0.003	0.520	0.431	0.198	0.064	0.016	0.003	0.001	0.000	0.000	0.000	0.005
2.5	-0.048	0.497	0.446	0.217	0.074	0.020	0.004	0.001	0.000	0.000	0.000	-0.097
2.6	-0.097	0.471	0.459	0.235	0.084	0.023	0.005	0.001	0.000	0.000	0.000	-0.206
2.7	-0.142	0.442	0.470	0.254	0.095	0.027	0.006	0.001	0.000	0.000	0.000	-0.323
2.8	-0.185	0.410	0.478	0.273	0.107	0.032	0.008	0.002	0.000	0.000	0.000	-0.452
2.9	-0.224	0.375	0.483	0.291	0.119	0.037	0.010	0.002	0.000	0.000	0.000	-0.597
3.0	-0.260	0.339	0.486	0.309	0.132	0.043	0.011	0.003	0.000	0.000	0.000	-0.767
3.1	-0.292	0.301	0.486	0.326	0.146	0.049	0.014	0.003	0.001	0.000	0.000	-0.971
3.2	-0.320	0.261	0.484	0.343	0.160	0.056	0.016	0.004	0.001	0.000	0.000	-1.225
3.3	-0.344	0.221	0.478	0.359	0.174	0.064	0.019	0.005	0.001	0.000	0.000	-1.560
3.4	-0.364	0.179	0.470	0.373	0.189	0.072	0.022	0.006	0.001	0.000	0.000	-2.033
3.5	-0.380	0.137	0.459	0.387	0.204	0.080	0.025	0.007	0.002	0.000	0.000	-2.767
3.6	-0.392	0.095	0.445	0.399	0.220	0.090	0.029	0.008	0.002	0.000	0.000	-4.104
3.7	-0.399	0.054	0.428	0.409	0.235	0.099	0.034	0.009	0.002	0.000	0.000	-7.416
3.8	-0.403	0.013	0.409	0.418	0.251	0.110	0.038	0.011	0.003	0.001	0.000	-31,40
3.9	-0.402	-0.027	0.388	0.425	0.266	0.121	0.043	0.013	0.003	0.001	0.000	14,75
4.0	-0.397	-0.066	0.364	0.430	0.281	0.132	0.049	0.015	0.004	0.001	0.000	6.013
4.1	-0.389	-0.103	0.338	0.433	0.296	0.144	0.055	0.018	0.005	0.001	0.000	3.764
4.2	-0.377	-0.139	0.311	0.434	0.310	0.156	0.062	0.020	0.006	0.001	0.000	2.716
4.3	-0.361	-0.172	0.281	0.433	0.324	0.169	0.069	0.023	0.007	0.002	0.000	2.100
4.4	-0.342	-0.203	0.250	0.430	0.336	0.182	0.076	0.026	0.008	0.002	0.000	1.688
4.5	-0.321	-0.231	0.218	0.425	0.348	0.195	0.084	0.030	0.009	0.002	0.001	1.387
4.6	-0.296	-0.257	0.185	0.417	0.359	0.208	0.093	0.034	0.011	0.003	0.001	1.154
4.7	-0.269	-0.279	0.151	0.407	0.369	0.221	0.102	0.038	0.012	0.003	0.001	0.965
4.8	-0.240	-0.298	0.116	0.395	0.378	0.235	0.111	0.043	0.014	0.004	0.001	0.805
4.9	-0.210	-0.315	0.081	0.381	0.385	0.248	0.121	0.048	0.016	0.005	0.001	0.666

Funciones de Bessel de primer tipo												
beta	**J0**	**J1**	**J2**	**J3**	**J4**	**J5**	**J6**	**J7**	**J8**	**J9**	**J10**	**J0/J1**
5.0	-0.178	-0.328	0.047	0.365	0.391	0.261	0.131	0.053	0.018	0.006	0.001	0.542
5.1	-0.144	-0.337	0.012	0.347	0.396	0.274	0.142	0.059	0.021	0.006	0.002	0.428
5.2	-0.110	-0.343	-0.022	0.327	0.398	0.287	0.153	0.065	0.024	0.007	0.002	0.321
5.3	-0.076	-0.346	-0.055	0.305	0.400	0.299	0.164	0.072	0.027	0.009	0.002	0.219
5.4	-0.041	-0.345	-0.087	0.281	0.399	0.310	0.175	0.079	0.030	0.010	0.003	0.119
5.5	-0.007	-0.341	-0.117	0.256	0.397	0.321	0.187	0.087	0.034	0.011	0.003	0.020
5.6	0.027	-0.334	-0.146	0.230	0.393	0.331	0.199	0.094	0.038	0.013	0.004	-0.081
5.7	0.060	-0.324	-0.174	0.202	0.387	0.340	0.210	0.103	0.042	0.015	0.005	-0.185
5.8	0.092	-0.311	-0.199	0.174	0.379	0.349	0.222	0.111	0.046	0.017	0.005	-0.295
5.9	0.122	-0.295	-0.222	0.145	0.369	0.356	0.234	0.120	0.051	0.019	0.006	-0.413
6.0	0.151	-0.277	-0.243	0.115	0.358	0.362	0.246	0.130	0.057	0.021	0.007	-0.544
6.1	0.177	-0.256	-0.261	0.085	0.344	0.367	0.257	0.139	0.062	0.024	0.008	-0.693
6.2	0.202	-0.233	-0.277	0.054	0.329	0.371	0.269	0.149	0.068	0.027	0.009	-0.866
6.3	0.224	-0.208	-0.290	0.024	0.313	0.373	0.279	0.159	0.074	0.030	0.010	-1.076
6.4	0.243	-0.182	-0.300	-0.006	0.295	0.374	0.290	0.170	0.081	0.033	0.012	-1.340
6.5	0.260	-0.154	-0.307	-0.035	0.275	0.374	0.300	0.180	0.088	0.037	0.013	-1.691
6.6	0.274	-0.125	-0.312	-0.064	0.254	0.372	0.309	0.191	0.095	0.040	0.015	-2.193
6.7	0.285	-0.095	-0.314	-0.092	0.231	0.368	0.318	0.201	0.103	0.045	0.017	-2.990
6.8	0.293	-0.065	-0.312	-0.118	0.208	0.363	0.326	0.212	0.111	0.049	0.019	-4.494
6.9	0.298	-0.035	-0.308	-0.144	0.183	0.356	0.333	0.223	0.119	0.054	0.021	-8.541
7.0	0.300	-0.005	-0.301	-0.168	0.158	0.348	0.339	0.234	0.128	0.059	0.024	-64,08
7.1	0.299	0.025	-0.292	-0.190	0.132	0.338	0.344	0.244	0.137	0.064	0.026	#####
7.2	0.295	0.054	-0.280	-0.210	0.105	0.327	0.349	0.254	0.146	0.070	0.029	5.431
7.3	0.288	0.083	-0.266	-0.228	0.078	0.314	0.352	0.264	0.155	0.076	0.032	3.491
7.4	0.279	0.110	-0.249	-0.244	0.051	0.299	0.353	0.274	0.165	0.082	0.035	2.541
7.5	0.266	0.135	-0.230	-0.258	0.024	0.283	0.354	0.283	0.174	0.089	0.039	1.969
7.6	0.252	0.159	-0.210	-0.270	-0.003	0.266	0.354	0.292	0.184	0.096	0.043	1.580
7.7	0.235	0.181	-0.187	-0.279	-0.030	0.248	0.352	0.300	0.194	0.103	0.047	1.294
7.8	0.215	0.201	-0.164	-0.285	-0.056	0.228	0.348	0.308	0.204	0.111	0.051	1.070
7.9	0.194	0.219	-0.139	-0.289	-0.081	0.207	0.344	0.314	0.214	0.118	0.056	0.887
8.0	0.172	0.235	-0.113	-0.291	-0.105	0.186	0.338	0.321	0.223	0.126	0.061	0.732
8.1	0.148	0.248	-0.086	-0.290	-0.129	0.163	0.330	0.326	0.233	0.135	0.066	0.596
8.2	0.122	0.258	-0.059	-0.287	-0.151	0.140	0.321	0.330	0.243	0.143	0.071	0.474
8.3	0.096	0.266	-0.032	-0.281	-0.171	0.116	0.311	0.334	0.252	0.152	0.077	0.361
8.4	0.069	0.271	-0.005	-0.273	-0.190	0.092	0.300	0.336	0.261	0.160	0.083	0.255
8.5	0.042	0.273	0.022	-0.263	-0.208	0.067	0.287	0.338	0.269	0.169	0.089	0.154
8.6	0.015	0.273	0.049	-0.250	-0.223	0.042	0.273	0.338	0.278	0.178	0.096	0.054
8.7	-0.013	0.270	0.075	-0.235	-0.237	0.018	0.257	0.337	0.285	0.188	0.103	-0.046
8.8	-0.039	0.264	0.099	-0.219	-0.249	-0.007	0.241	0.335	0.292	0.197	0.110	-0.149
8.9	-0.065	0.256	0.123	-0.201	-0.258	-0.031	0.223	0.332	0.299	0.206	0.117	-0.255
9.0	-0.090	0.245	0.145	-0.181	-0.265	-0.055	0.204	0.327	0.305	0.215	0.125	-0.368
9.1	-0.114	0.232	0.165	-0.160	-0.271	-0.078	0.185	0.322	0.310	0.224	0.132	-0.491
9.2	-0.137	0.217	0.184	-0.137	-0.274	-0.101	0.164	0.315	0.315	0.233	0.140	-0.629
9.3	-0.158	0.200	0.201	-0.114	-0.274	-0.122	0.143	0.307	0.319	0.241	0.148	-0.787
9.4	-0.177	0.182	0.215	-0.090	-0.273	-0.142	0.122	0.297	0.321	0.250	0.157	-0.973
9.5	-0.194	0.161	0.228	-0.065	-0.269	-0.161	0.099	0.287	0.323	0.258	0.165	-1.203
9.6	-0.209	0.140	0.238	-0.040	-0.263	-0.179	0.077	0.275	0.324	0.265	0.173	-1.498
9.7	-0.222	0.117	0.246	-0.015	-0.255	-0.195	0.054	0.262	0.324	0.273	0.182	-1.902
9.8	-0.232	0.093	0.251	0.010	-0.245	-0.210	0.031	0.248	0.323	0.280	0.190	-2.502
9.9	-0.240	0.068	0.254	0.034	-0.233	-0.223	0.008	0.233	0.321	0.286	0.199	-3.515
10.0	-0.246	0.043	0.255	0.058	-0.220	-0.234	-0.014	0.217	0.318	0.292	0.207	-5.657

Ceros de la Función de Bessel

Tabla C.2: Ceros de las funciones de Bessel: valores de β cuando $J_n(\beta)=0$

	Orden de la función de Bessel, n						
	0	1	2	3	4	5	6
β para el primer cero	2.40	3.83	5.14	6.38	7.59	8.77	9.93
β para el segundo cero	5.52	7.02	8.42	9.76	11.06	12.34	13.59
β para el tercer cero	8.65	10.17	11.62	13.02	14.37	15.70	17.00
β para el cuarto cero	11.79	13.32	14.80	16.22	17.62	18.98	20.32
β para el quinto cero	14.93	16.47	17.96	19.41	20.83	22.21	25.59
β para el sexto cero	18.07	19.61	21.12	22.58	24.02	25.43	26.82
β para el séptimo cero	21.21	22.76	24.27	25.75	27.20	28.63	30.03
β para el octavo cero	24.35	25.90	27.42	28.91	30.37	31.81	33.23

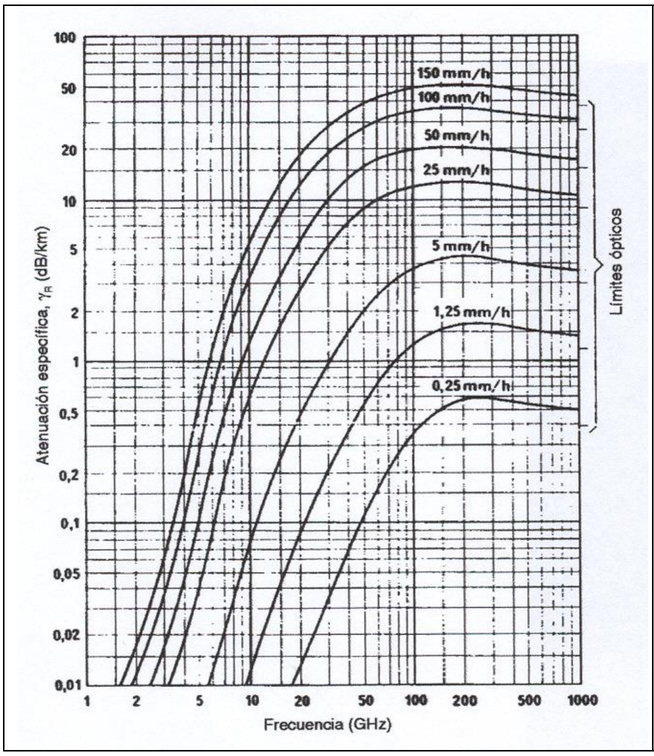
Appendix A: Fresnel Integrals

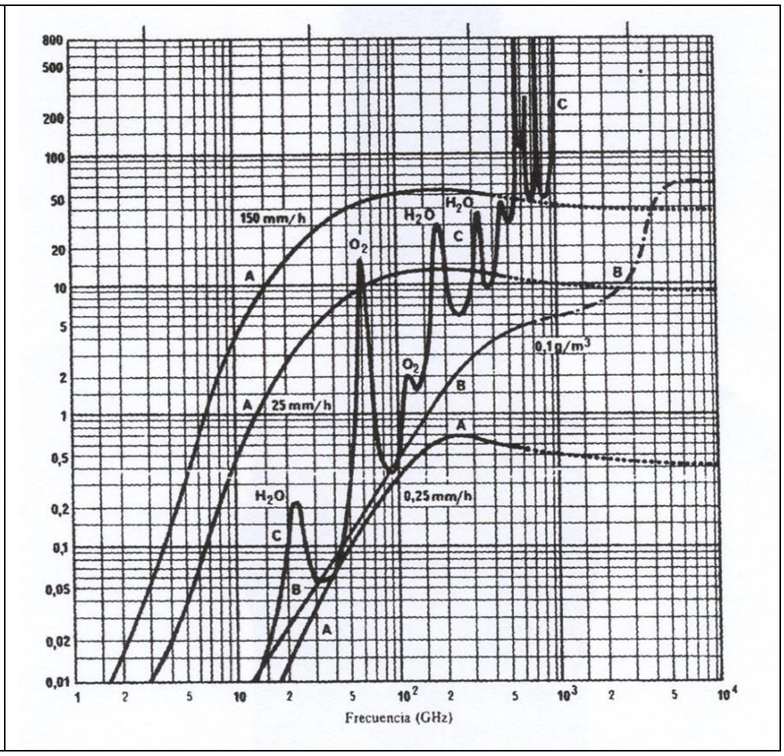
$$C(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right)dt$$

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right)dt$$

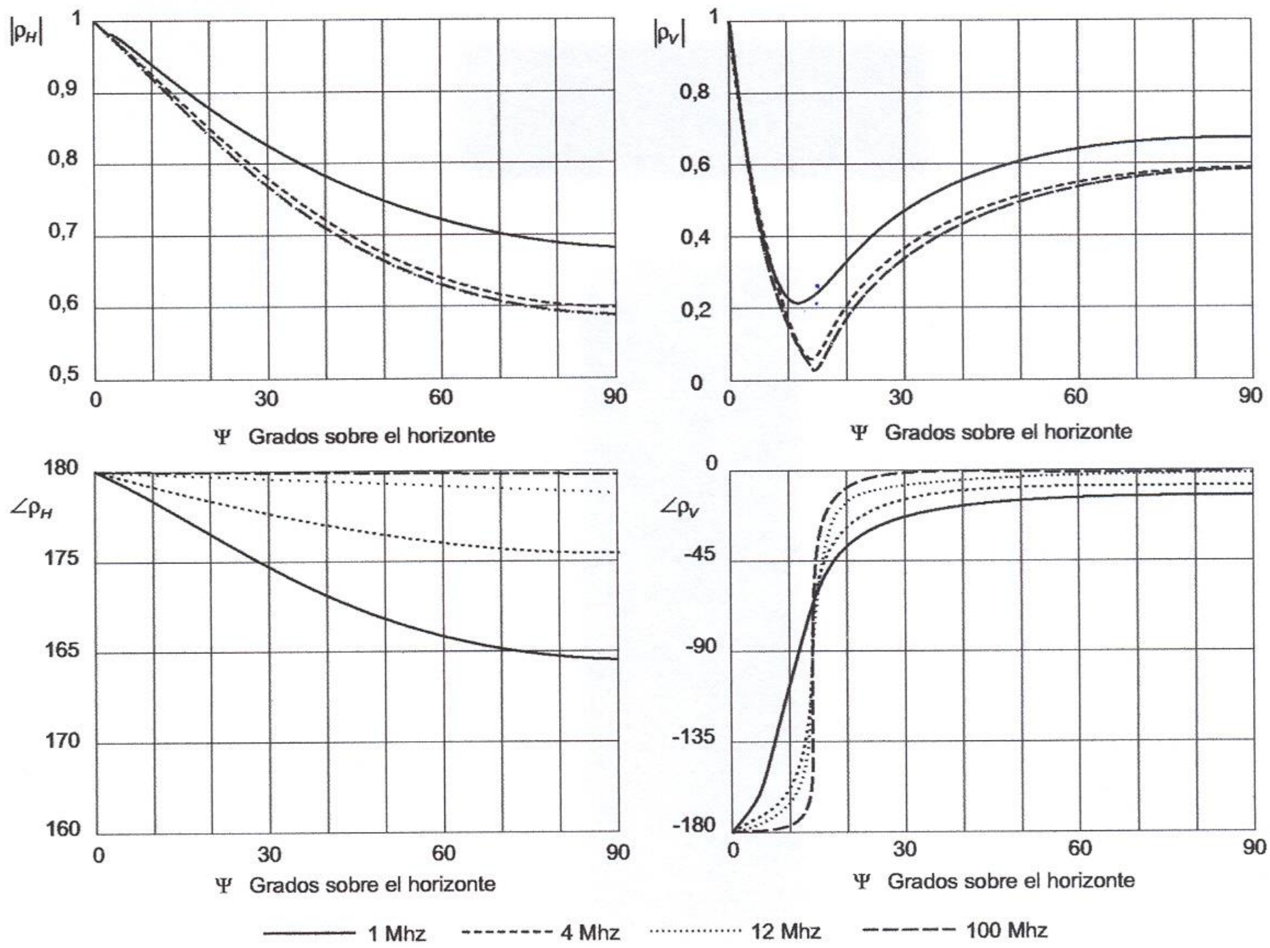
x	$C(x)$	$S(x)$	x	$C(x)$	$S(x)$
0	0	0	1.0500	0.7759	0.4880
0.0500	0.0500	0.0001	1.1000	0.7638	0.5365
0.1000	0.1000	0.0005	1.1500	0.7436	0.5821
0.1500	0.1500	0.0018	1.2000	0.7154	0.6234
0.2000	0.1999	0.0042	1.2500	0.6801	0.6587
0.2500	0.2498	0.0082	1.3000	0.6386	0.6863
0.3000	0.2994	0.0141	1.3500	0.5923	0.7050
0.3500	0.3487	0.0224	1.4000	0.5431	0.7135
0.4000	0.3975	0.0334	1.4500	0.4933	0.7111
0.4500	0.4455	0.0474	1.5000	0.4453	0.6975
0.5000	0.4923	0.0647	1.5500	0.4018	0.6731
0.5500	0.5377	0.0857	1.6000	0.3655	0.6389
0.6000	0.5811	0.1105	1.6500	0.3388	0.5968
0.6500	0.6219	0.1393	1.7000	0.3238	0.5492
0.7000	0.6597	0.1721	1.7500	0.3219	0.4994
0.7500	0.6935	0.2089	1.8000	0.3336	0.4509
0.8000	0.7228	0.2493	1.8500	0.3584	0.4077
0.8500	0.7469	0.2932	1.9000	0.3945	0.3733
0.9000	0.7648	0.3398	1.9500	0.4391	0.3511
0.9500	0.7760	0.3885	2.0000	0.4883	0.3434
1.0000	0.7799	0.4383	2.0500	0.5374	0.3513

Gráficas del tema de propagación en entorno natural

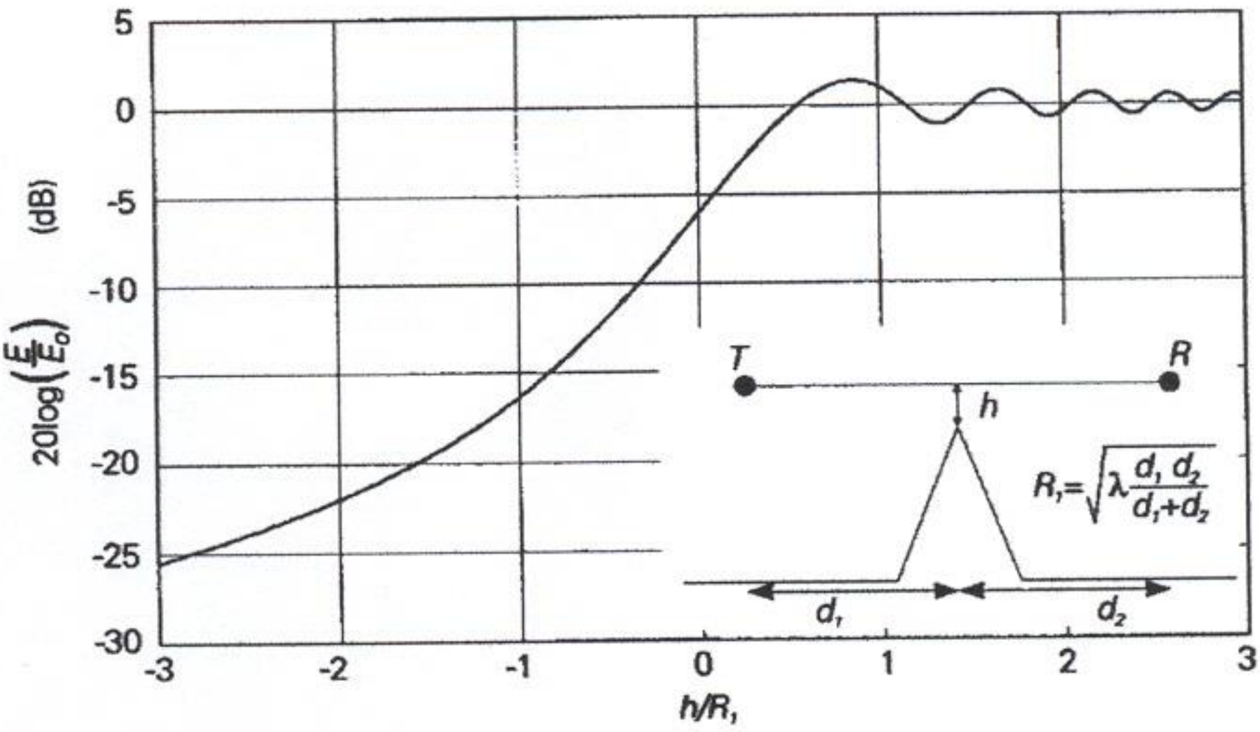




Coeficiente de reflexión para polarización horizontal y vertical frente al ángulo de elevación



Difracción por filo de cuchillo



Propagación por onda de superficie

FIGURA 1

Curvas de propagación de la onda de superficie; agua del mar, salinidad baja, σ = 1 S/m, ε = 80

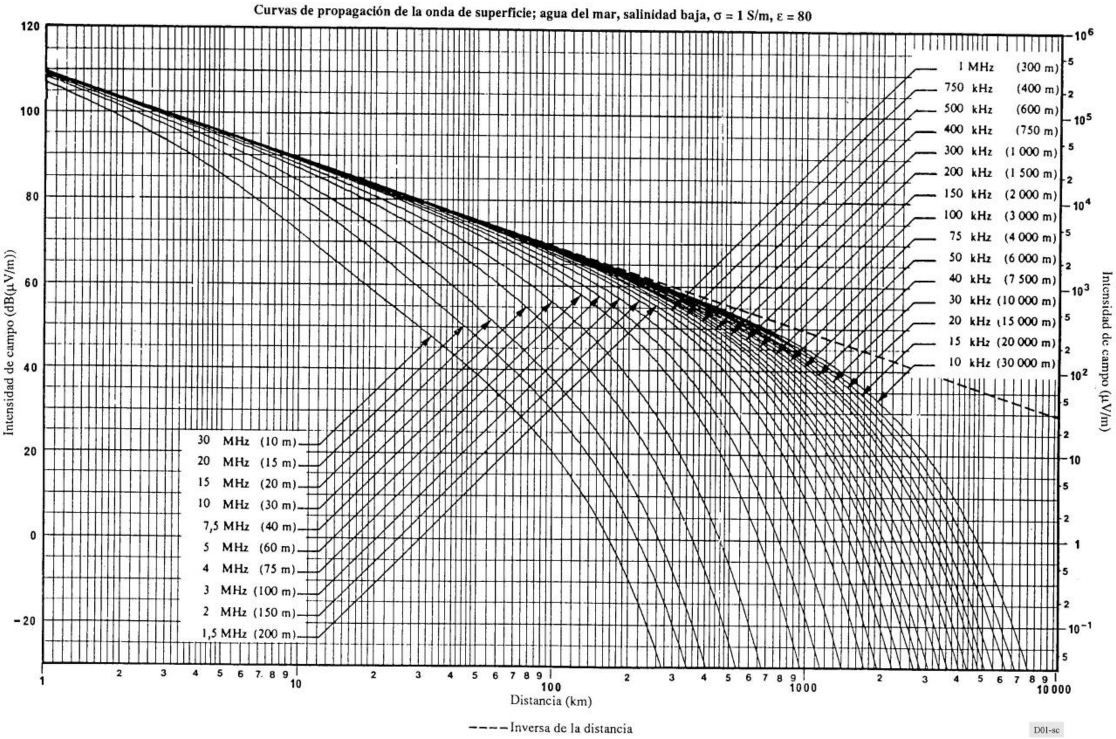


FIGURA 2

Curvas de propagación de la onda de superficie; agua del mar, salinidad media, σ = 5 S/m, ε = 70

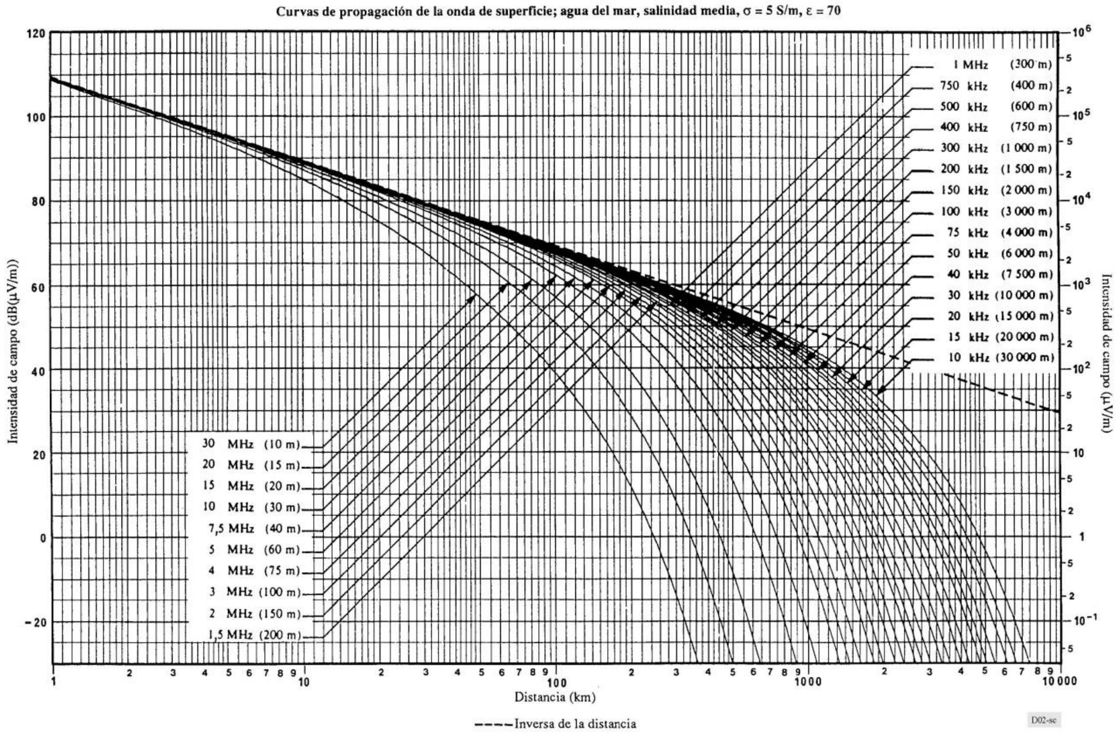
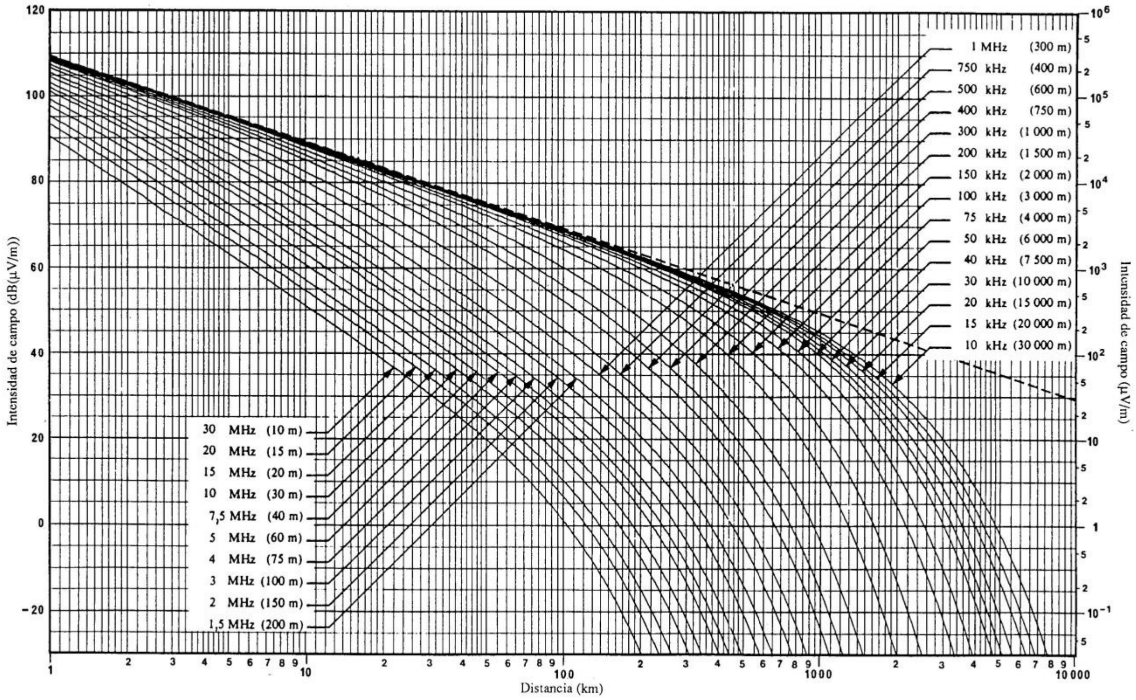


FIGURA 3

Curvas de propagación de la onda de superficie; agua dulce, $\sigma = 3 \times 10^{-3}$ S/m, $\varepsilon = 80$

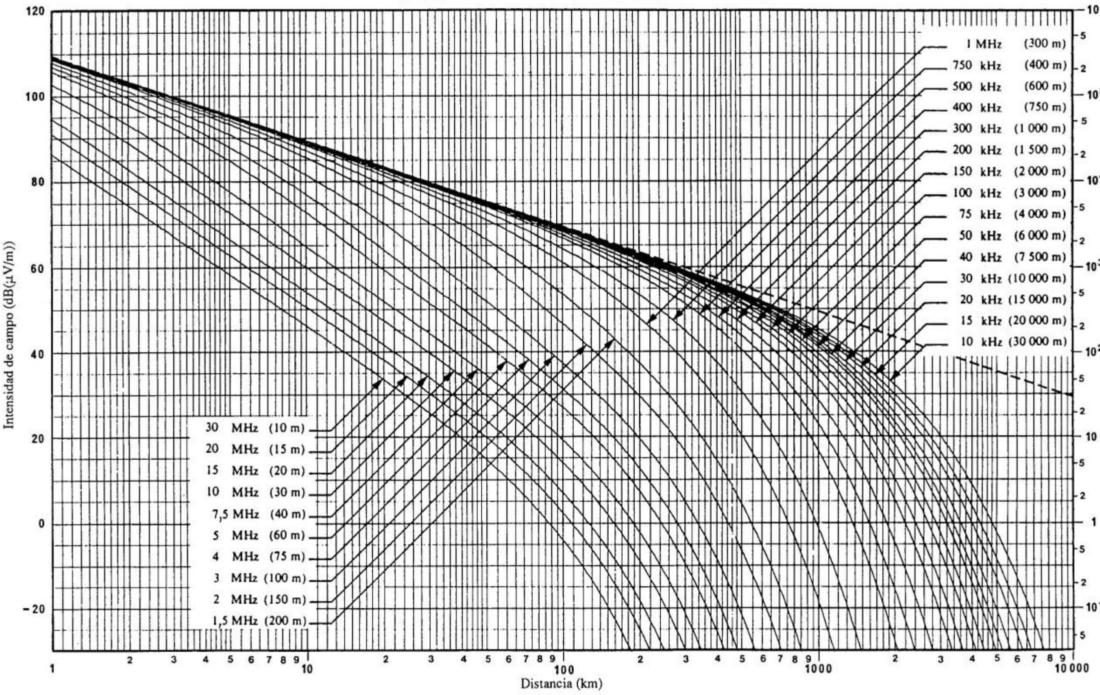


– – – –Inversa de la distancia

D03-sc

FIGURA 4

Curvas de propagación de la onda de superficie; tierra, σ = 3 × 10–2 S/m, ε = 40

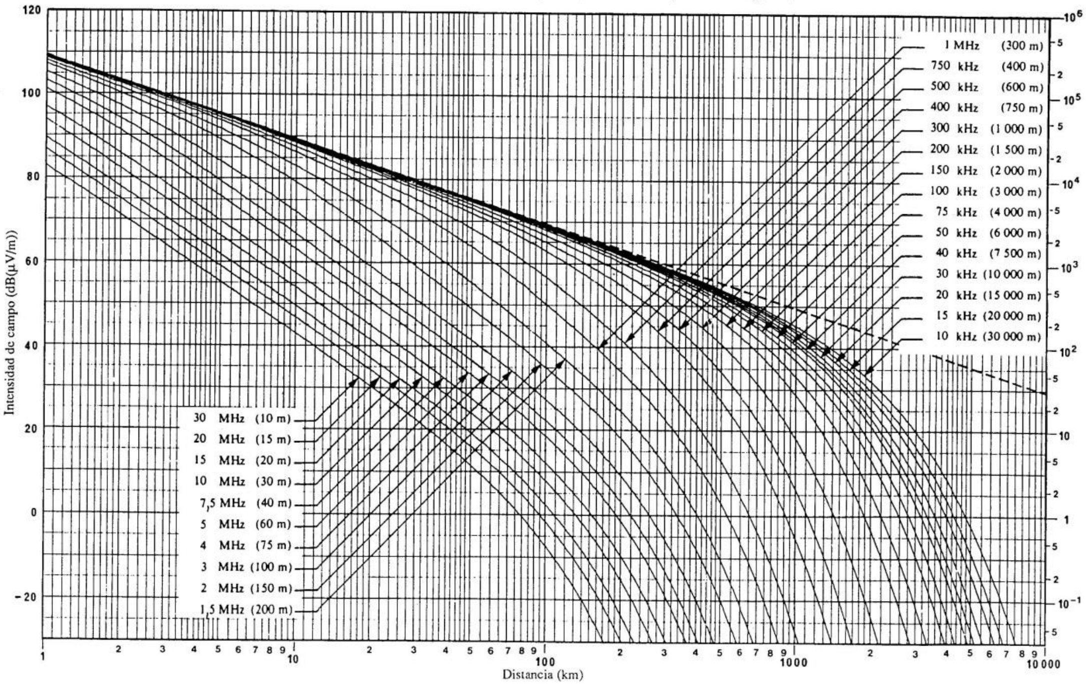


– – – – Inversa de la distancia

D04-sc

FIGURA 5

Curvas de propagación de la onda de superficie; tierra húmeda, σ = 10–2 S/m, ε = 30



– – – – Inversa de la distancia

D05-sc

FIGURA 6

Curvas de propagación de la onda de superficie; tierra, σ = 3 × 10^{-3} S/m, ε = 22

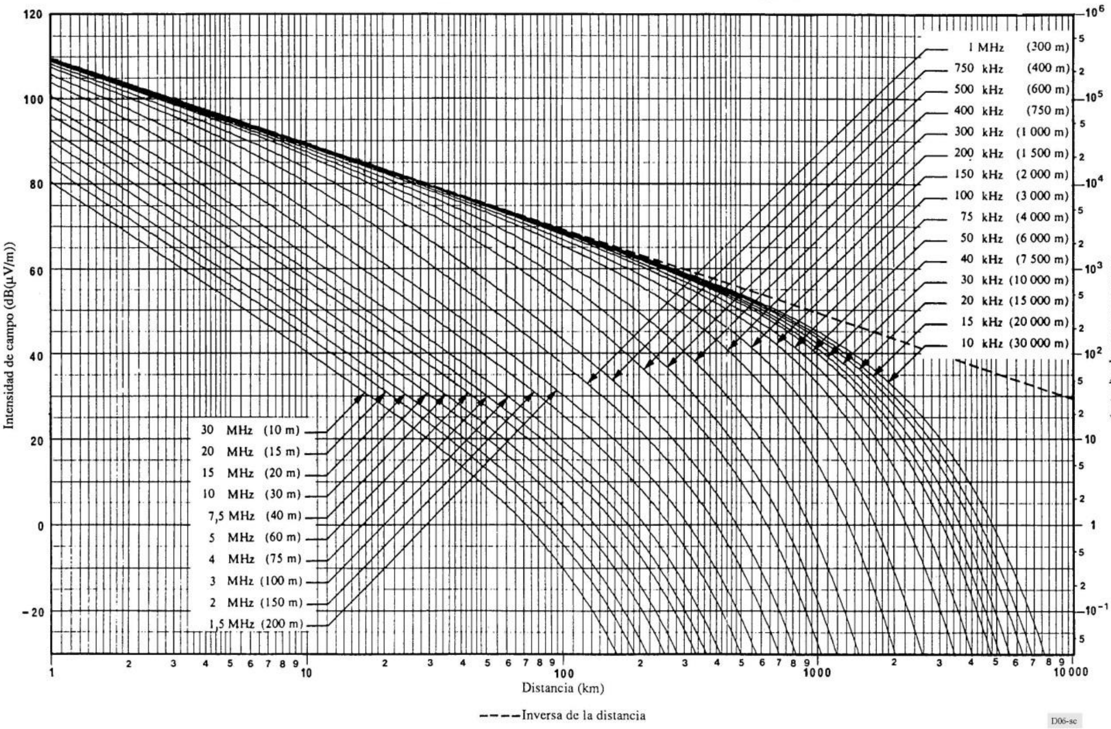


FIGURA 7

Curvas de propagación de la onda de superficie; tierra moderadamente seca, σ = 10^{-3} S/m, ε = 15

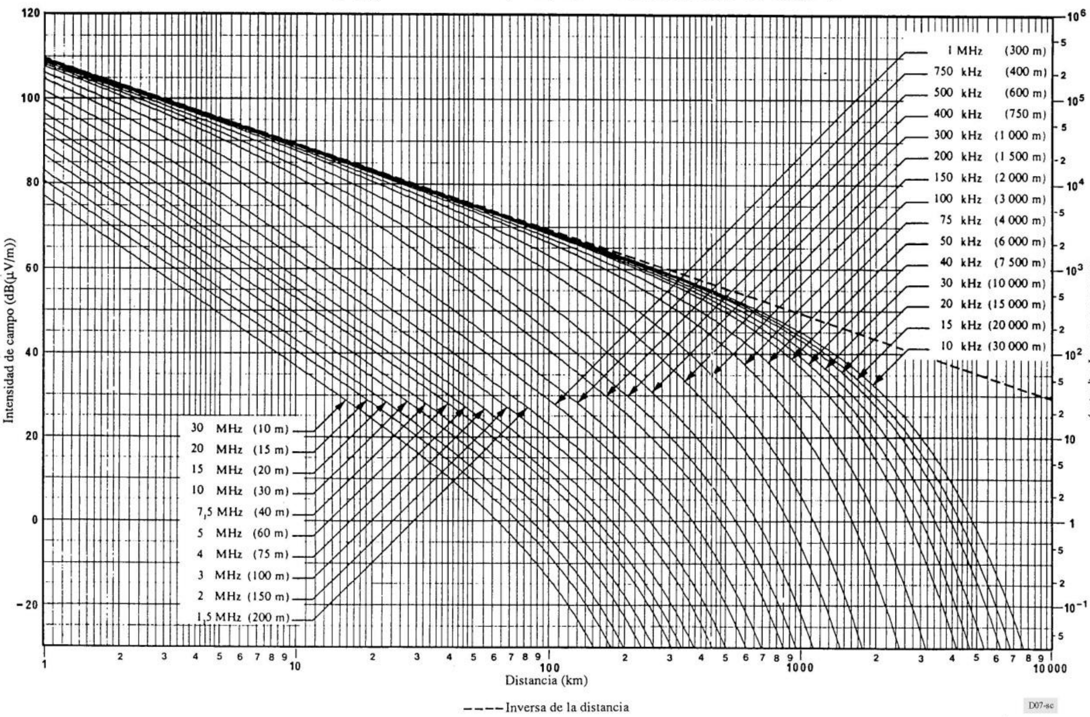
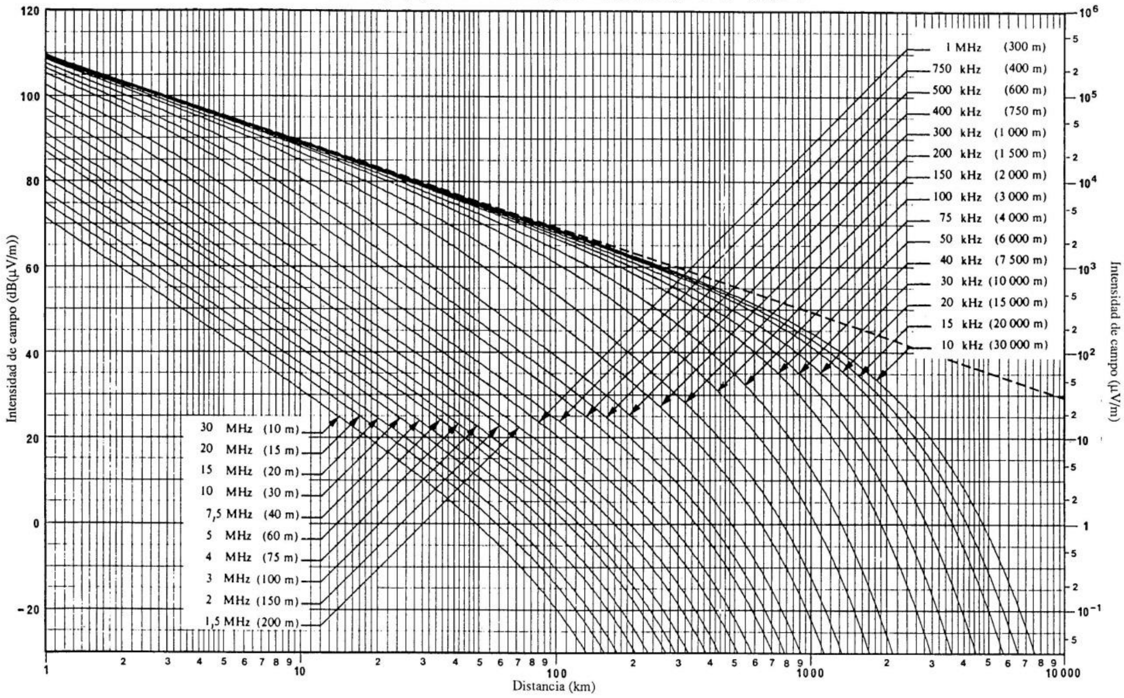


FIGURA 8

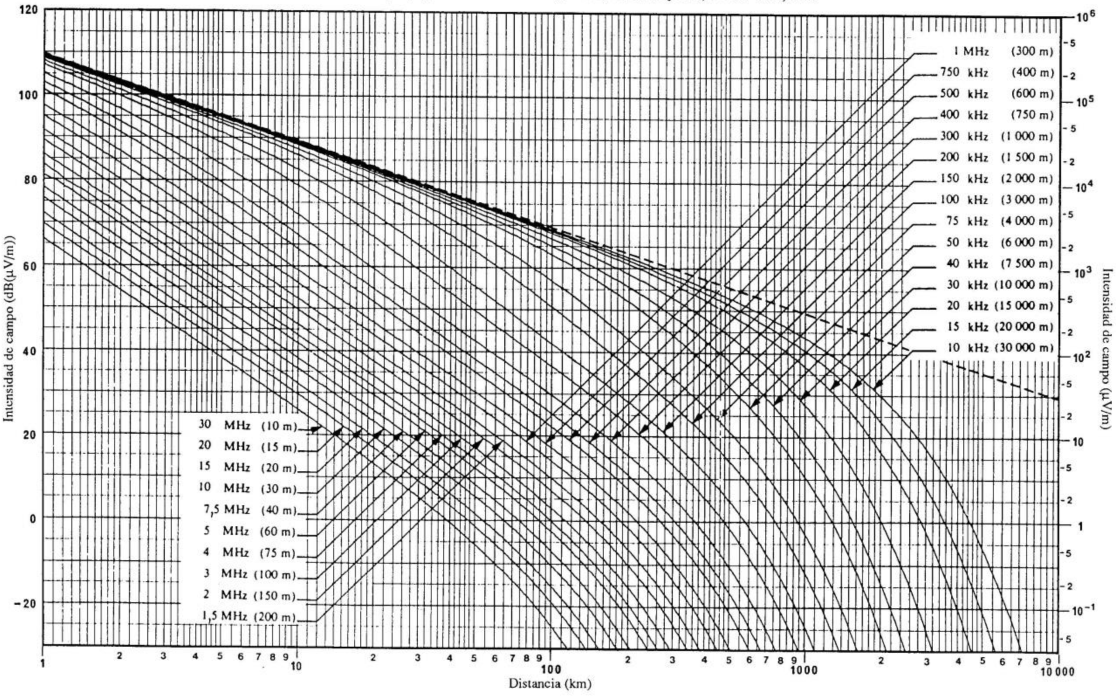
Curvas de propagación de la onda de superficie; tierra seca, $\sigma = 3 \times 10^{-4}$ S/m, $\varepsilon = 7$



----Inversa de la distancia

FIGURA 9

Curvas de propagación de la onda de superficie; tierra muy seca, $\sigma = 10^{-4}$ S/m, $\varepsilon = 3$



----Inversa de la distancia

FIGURA 10

Curvas de propagación de la onda de superficie; hielo de agua dulce, –1 °C, σ = 3 × 10–5 S/m, ε = 3

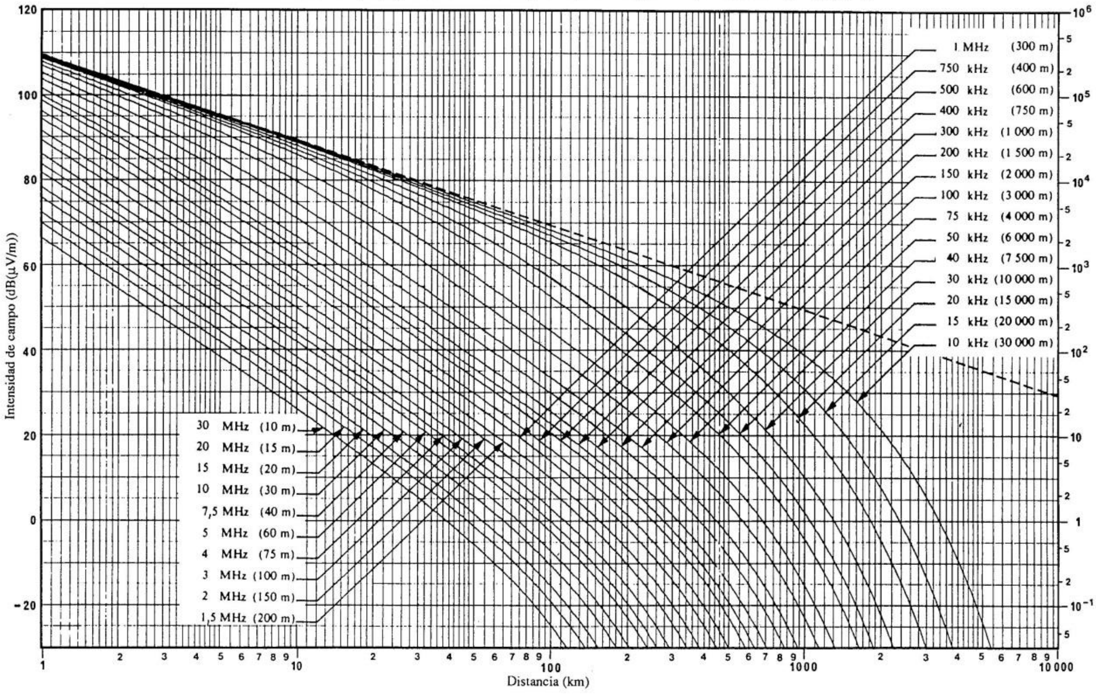


FIGURA 11

Curvas de propagación de la onda de superficie; hielo de agua dulce, –10 °C, σ = –10–5 S/m, ε = 3

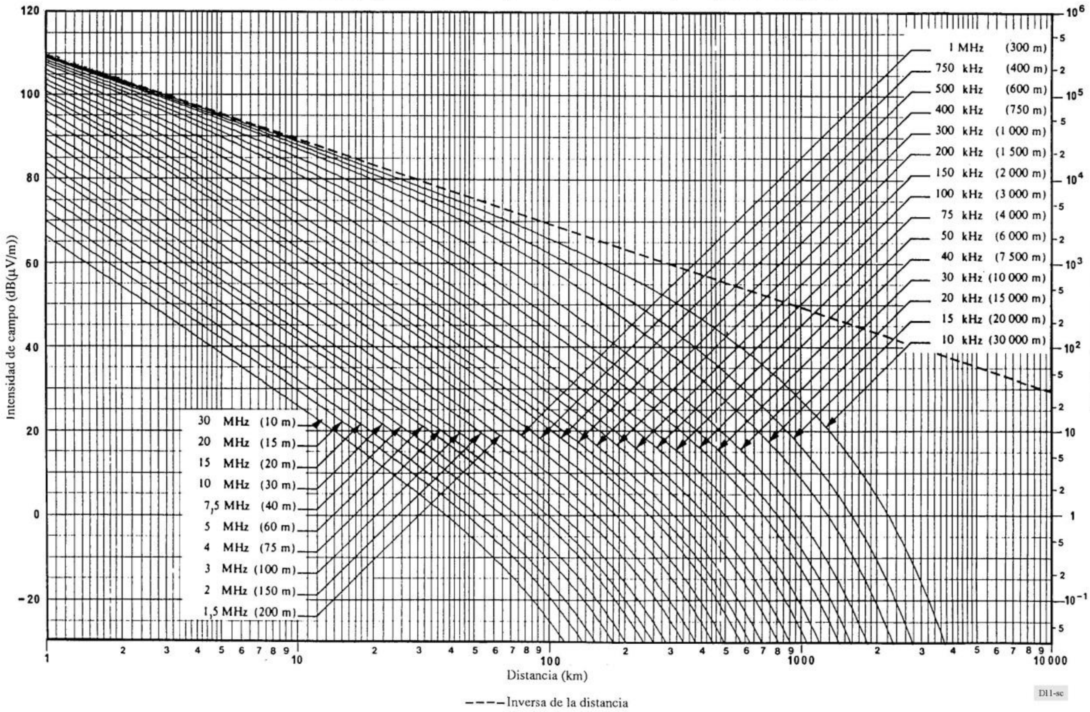
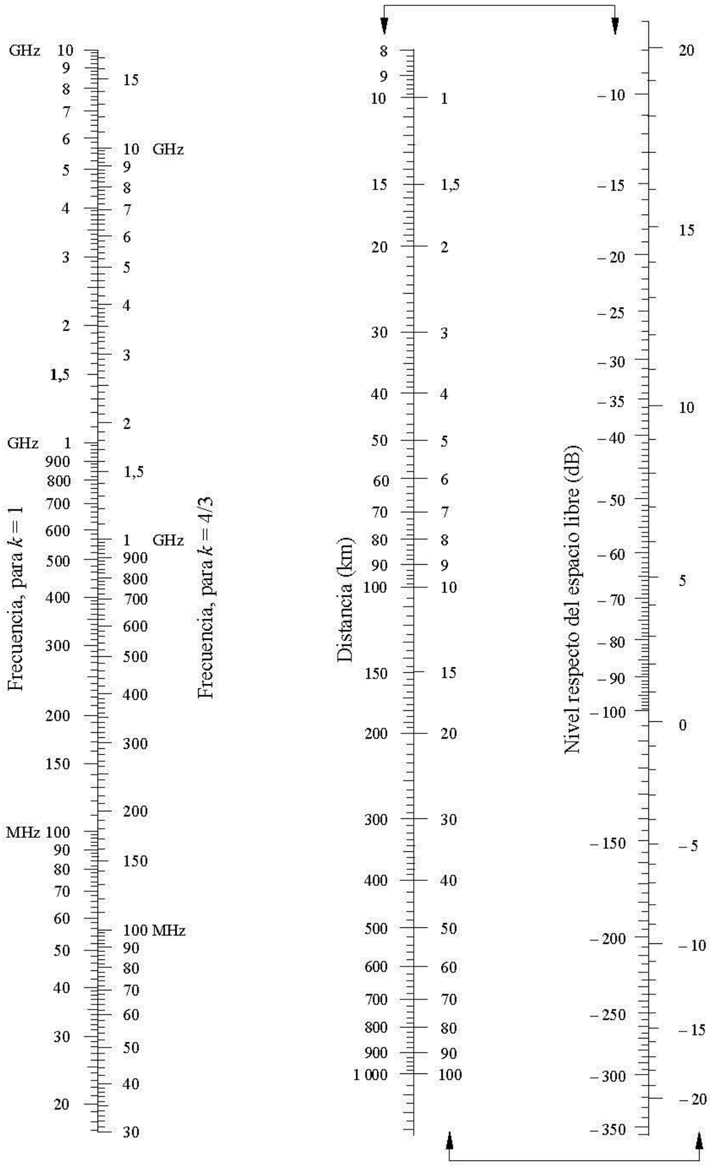


FIGURA 2

Difracción en una tierra esférica – Efecto de la distancia



Polarización horizontal sobre tierra y mar
Polarización vertical sobre tierra

(Las escalas unidas por flechas han de utilizarse conjuntamente)

0526-02

FIGURA 3

Difracción en una tierra esférica – Ganancia de altura

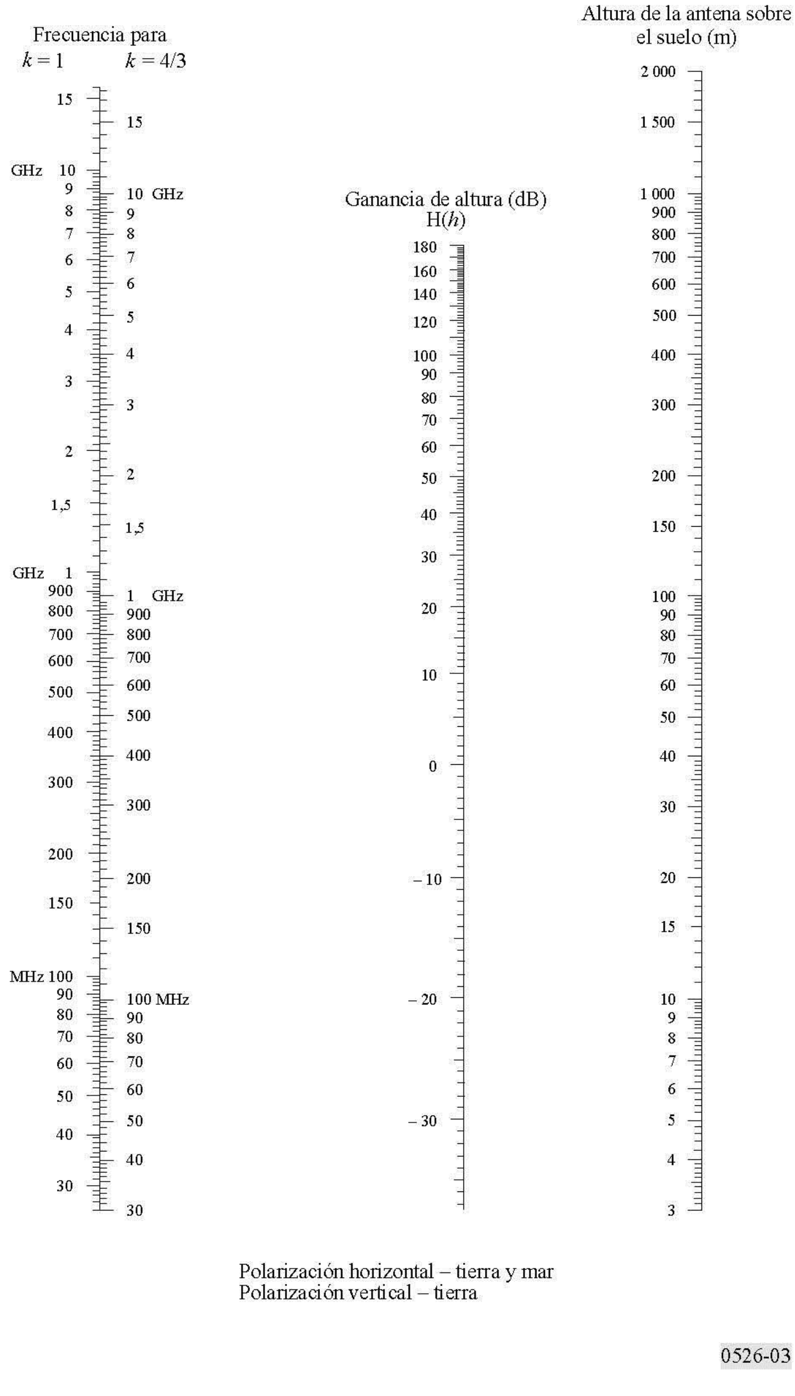
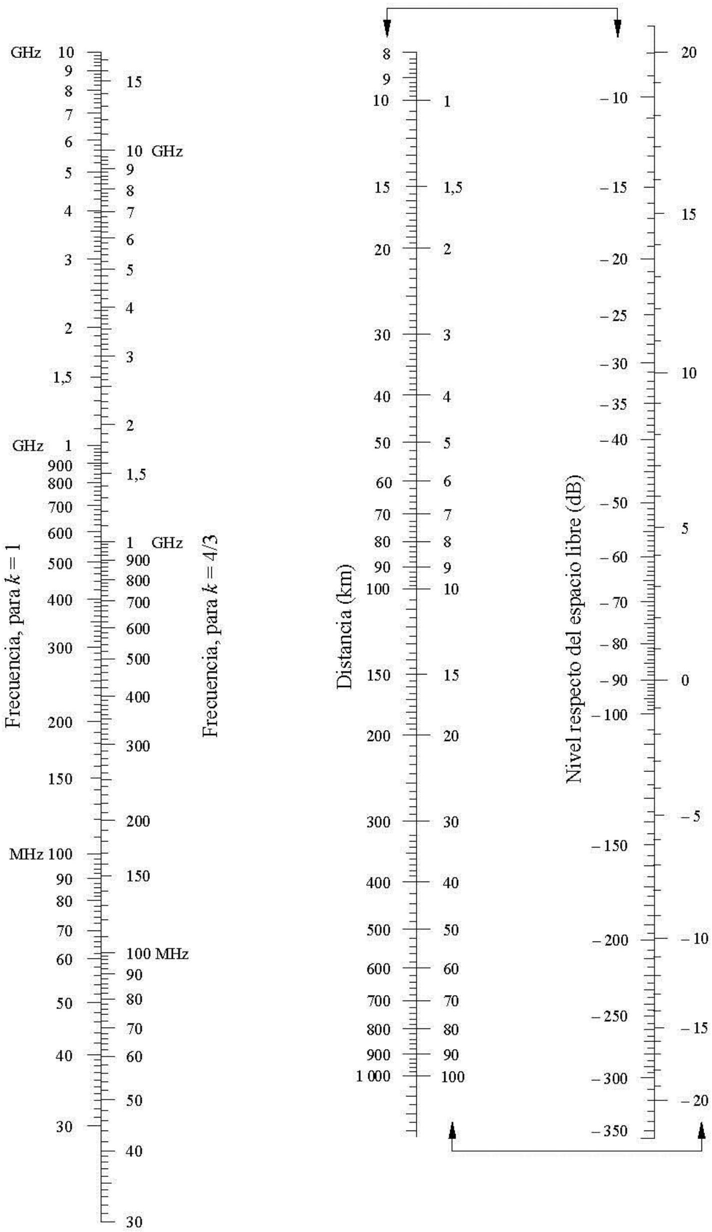


FIGURA 4

Difracción en una tierra esférica – Efecto de la distancia

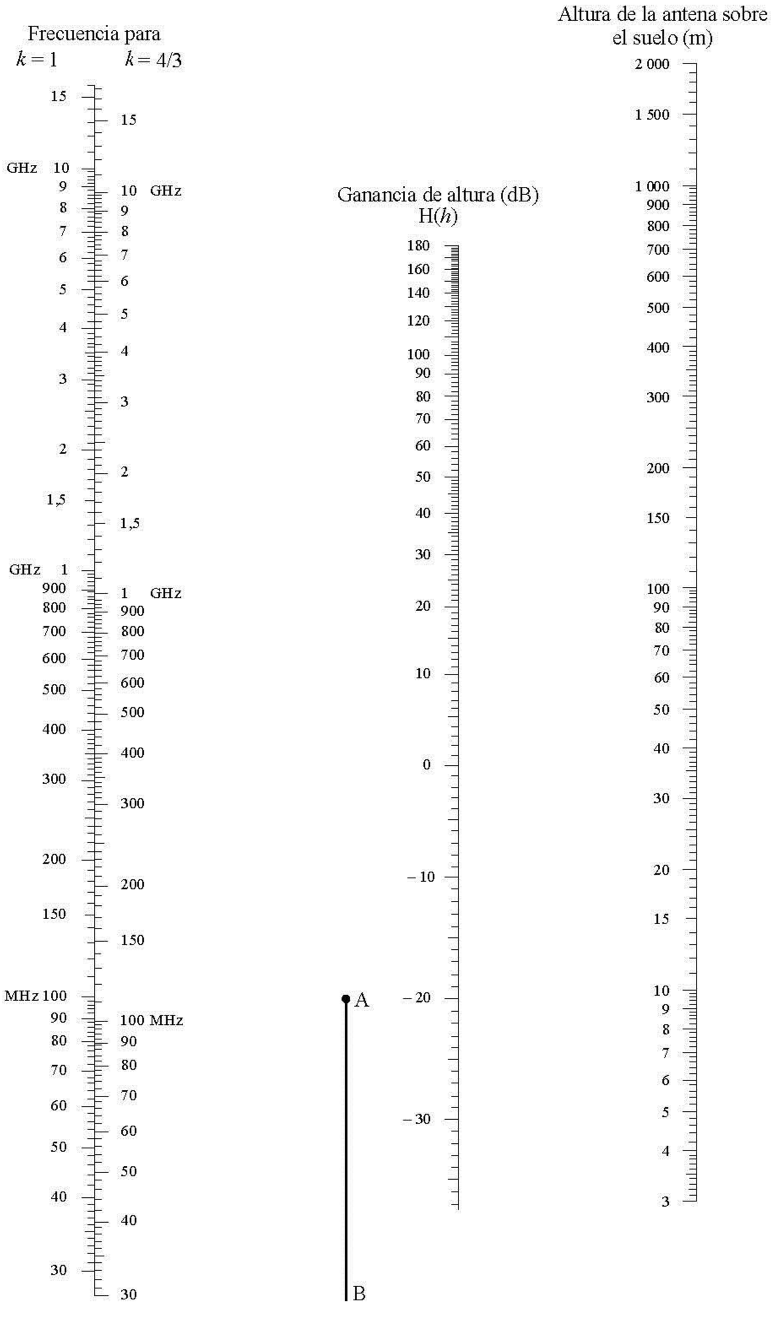


Polarización vertical sobre el mar
(Las escalas unidas por flechas han de utilizarse conjuntamente)

0526-04

FIGURA 5

Difracción en una tierra esférica – Ganancia de altura



Polarización vertical – mar

0526-05